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Department of PHYSICS

I B.Sc. SEMESTER-II

PAPER II: WAVE OPTICS

STUDY MATERIAL

Name of the Student : ________________________________

Roll Number : ________________________________

Group : ________________________________

Academic Year : ________________________________

Teaching Faculty

Dr. KHDHIRBRAHMENDRA V

G.M. MURALI KRISHNA

B.Sc. PHYSICS SYLLABUS UNDER CBCS

[2020-21 Batch onwards]

I Year B.Sc.-Physics: II Semester

Course-II: WAVE OPTICS

Work load: 60 hrs per semester **4 hrs/week**

UNIT-I Interference of light: (12hrs)

Introduction, Conditions for interference of light, Interference of light by division of wave front and amplitude, Phase change on reflection- Stokes' treatment, Lloyd's single mirror, Interference in thin films: Plane parallel and wedge- shaped films, colours in thin films, Newton's rings in reflected light-Theory and experiment, Determination of wavelength of monochromatic light, Michelson interferometer and determination of wavelength.

UNIT-II Diffraction of light: (12hrs)

Introduction, Types of diffraction: Fresnel and Fraunhoffer diffractions, Distinction between Fresnel and Fraunhoffer diffraction, Fraunhoffer diffraction at a single slit, Plane diffraction grating, Determination of wavelength of light using diffraction grating, Resolving power of grating, Fresnel's half period zones, Explanation of rectilinear propagation of light, Zone plate, comparison of zone plate with convex lens.

UNIT-III Polarisation of light: (12hrs)

Polarized light: Methods of production of plane polarized light, Double refraction, Brewster's law, Malus law, Nicol prism, Nicol prism as polarizer and analyzer, Quarter wave plate, Half wave plate, Plane, Circularly and Elliptically polarized light-Production and detection, Optical activity, Laurent's half shade polarimeter: determination of specific rotation, Basic principle of LCDs

UNIT-IV Aberrations and Fibre Optics: (12hrs)

Monochromatic aberrations, Spherical aberration, Methods of minimizing spherical aberration, Coma, Astigmatism and Curvature of field, Distortion; Chromatic aberration-the achromatic doublet; Achromatism for two lenses (i) in contact and (ii) separated by a distance.

Fibre optics: Introduction to Fibers, different types of fibers, rays and modes in an optical fiber, Principles of fiber communication (qualitative treatment only), Advantages of fiber optic communication.

UNIT-V Lasers and Holography: (12hrs)

Lasers: Introduction, Spontaneous emission, stimulated emission, Population Inversion, Laser principle, Einstein coefficients, Types of lasers-He-Ne laser, Ruby laser, Applications of lasers; Holography: Basic principle of holography, Applications of holography

Course outcomes

On successful completion of this course, the student will be able to:

- ✓ Understand the phenomenon of interference of light and its formation in (i) Lloyd's single mirror due to division of wave front and (ii) Thin films, Newton's rings and M Michelson interferometer due to division of amplitude.
- ✓ Distinguish between Fresnel's diffraction and Fraunhoffer diffraction and observe the diffraction patterns in the case of single slit and the diffraction grating.
- ✓ Describe the construction and working of zone plate and make the comparison of zone plate with convex lens.
- ✓ Explain the various methods of production of plane, circularly and polarized light and their detection and the concept of optical activity.
- ✓ Comprehend the basic principle of laser, the working of He-Ne laser and Ruby lasers and their applications in different fields.
- ✓ Explain about the different aberrations in lenses and discuss the methods of minimizing them.
- ✓ Understand the basic principles of fibre-optic communication and explore the field of Holography and Nonlinear optics and their applications.

UNIT-I
INTERFERENCE

1.1 PRINCIPLE OF SUPERPOSITION

This principle states that the resultant displacement of a particle of the medium acted upon by two or more waves simultaneously is the algebraic sum of the displacements of the same particle due to individual waves, in the absence of the others.

$$y = y_1 + y_2 + y_3 + y_4 \ldots$$

Suppose due to a single wave train the displacement of the particle at a certain point at any instant is y_1 in a given direction and that due to another wave train, the displacement is y_2 in the absence of the first.

According to the principle of superposition, the instantaneous resultant displacement R of the particle due to two waves acting together in same direction is expressed by

$$y = y_1 + y_2$$

If the two displacements are in opposite directions, then instantaneous resultant displacement

$$y = y_1 - y_2$$

1.2 INTERFERENCE OF LIGHT

When two light waves superimpose, the resultant amplitude/intensity in the region of superposition is different from the individual waves. This modification in the intensity distribution about region of superposition is called *interference*. When the resultant amplitude is the sum of the amplitudes due to two waves, the interference is known as *constructive interference* and when the resultant amplitude is equal to the difference of two amplitudes, the interference is known as *destructive interference*.

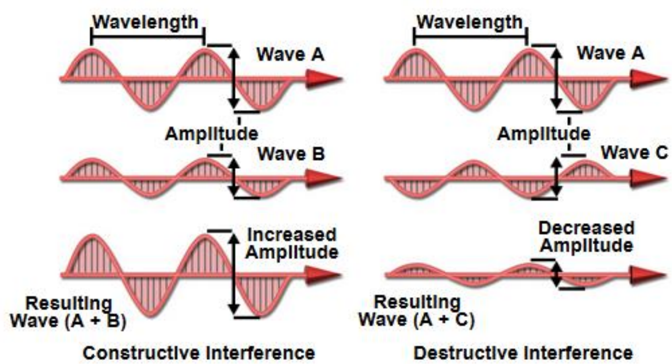


Fig 1.1

1.3 YOUNG'S EXPERIMENT

The first experimental demonstration of interference of light was given by Thomas Young in 1801. He allowed the sunlight to pass through a pin hole S and then at some distance two pin holes S_1 and S_2 are drilled in an opaque screen. The interference pattern was observed on a screen XY. He observed few colored bright and dark bands on the screen. Now-a-days, pin holes S_1 and S_2 are replaced by narrow slits and sunlight is replaced by monochromatic light. Now the interference pattern consists of equally spaced bright and dark bands as shown in Fig. 1.2.

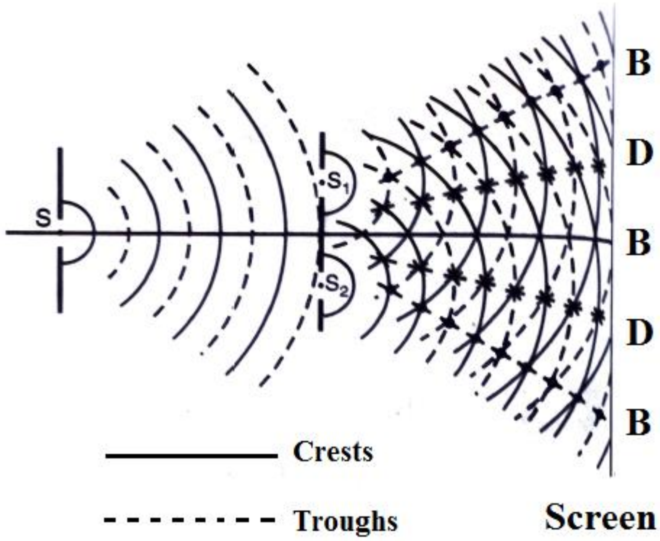


Fig. 1.2

Explanation on wave theory:

When sunlight passes through pin holes S, spherical waves are spread out. According to Huygen's principle, each point on the wavefront is a centre of secondary wavelets. Hence, spherical waves spread out from pin holes S_1 and S_2. The secondary wavelets are superimposed to form interference.

In the figure, wave crests are represented with continuous circular arcs and the wave troughs are indicated with dotted circular arcs. At points where a crest due to one wave falls on the crest due another wave (or trough on a trough), the resultant amplitude is the sum of the amplitudes due to each wave separately. This is known as constructive interference. At points, where a crest due to one wave falls on the trough of the other wave the resultant amplitude is equal to the differenced the amplitudes due to separate waves and hence the resultant intensity is minimum. This is known destructive interference. Thus, on the screen a number of alternate bright and dark regions of equal width called interference fringes, are observed.

Intensity at a Point in a plane:

Analytical treatment:

Let us suppose a_1 and a_2 are the amplitudes of interfering waves. Let Ø be the phase difference between the interfering waves. If Y_1 and Y_2 are their respective instantaneous displacements, then

$$Y_1 = a_1 \, sin\omega t \qquad (1)$$

$$Y_2 = a_2 \, sin\,(\omega t + \phi) \qquad (2)$$

According to super position principle, the resultant instantaneous displacement of the resultant wave is given by

$$Y = Y_1 + Y_2$$

$$Y = a_1 \, sin\omega t + a_2 \, sin\,(\omega t + \phi)$$

$$= a_1 sin\omega t + a_2 (sin\omega t cos\,\phi + cos\omega t sin\,\phi$$

$$= (a_1 + a_2 cos\,\phi)\, sin\omega t + a_2 sin\,\phi\, cos\omega t \qquad (3)$$

$$Put \;\; a_1 + a_2\, cos\,\phi = Rcos\theta \qquad (4)$$

$$a_2 \sin\phi = R\sin\theta$$

(5)

$$Y = R\cos\theta \sin\omega t + R\sin\theta\cos\omega t$$
$$= R\sin(\omega t + \theta)$$

Where R is the resultant amplitude of the wave

$$(1)2 + (2)2 \quad \Rightarrow \quad R^2 \cos^2\theta + R^2 \sin^2\theta = (a_1 + a_2\cos\phi)^2 + a_2^2 \sin^2\phi$$
$$\Rightarrow \quad R^2 (\cos^2\theta + \sin^2\theta) = a_1^2 + a_2^2 \cos^2\phi + 2a_1a_2\cos\phi + a_2^2 \sin^2\phi$$
$$\Rightarrow \quad R^2 = a_1^2 + 2\, a_1a_2\cos\phi + a_2^2 (\cos^2\phi + \sin^2\phi)$$
$$\Rightarrow \quad R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$$
$$R = \sqrt{a_1^2 + {a_2}^2 + 2a_1a_2\cos\phi}$$

For constructive interference, ϕ *= 0, 2π, 4π…..2nπ*

$$If\ \phi = 0, \quad \cos\phi = 1$$
$$R = \sqrt{(a_1 + a_2)^2}$$
$$If\ a_1 = a_1 = a \ \Rightarrow\ R = 2a$$

Resultant intensity $I = R^2$

$$= (2a)^2$$
$$= 4a^2$$
$$= 4I$$

For destructive interference, $\phi = \pi, 3\pi, 5\pi \ldots\ldots (2n+1)\pi$

$$If\ \phi = \pi, \quad \cos\phi = -1$$
$$R = \sqrt{a_1^2 + {a_2}^2 - 2a_1a_2}$$
$$= \sqrt{(a_1 - a_2)^2} = a_1 - a_2$$

If $a_1 = a_1 = a \ \Rightarrow\ R = 0$

Resultant intensity $I = R^2 = 0$

Energy Distribution:

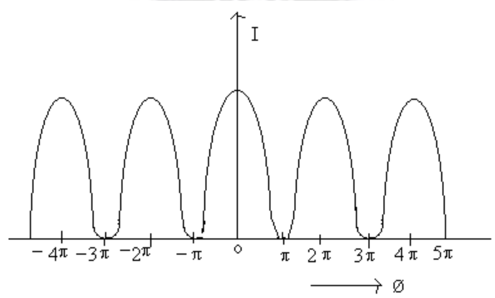


Fig. 1.3

Note

$R_{max} = a_1 + a_2$ $\quad I_{max} = (a_1 + a_2)^2 = R^2_{max}$

$R_{min} = a_1 - a_2$ $\quad I_{min} = (a_1 - a_2)^2 = R^2_{min}$

1.4 COHERENT SOURCES AND COHERENCE

The two sources which maintain zero or any constant phase relation between themselves are known as coherent sources. This phenomenon is known as coherence. However, if the phase difference changes with time, the two sources are known as incoherent sources.

Interference is not possible with light emitting from two different sources. *To produce coherent sources, the two sources should be derived from a single source by some suitable device.*

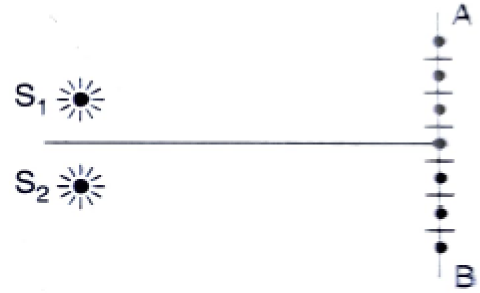


Fig. 1.4

Consider the sources S_1 and S_2 emitting light of particular wavelength (say λ). AB is a screen placed at some distance from the sources. The dots and dashes on the screen indicating the points with path difference $n\lambda$ and $(n+\frac{1}{2})\lambda$ respectively, where n is any integer.

Now if,

(1) **the waves from S_1 and S_2 are in same phase**. In this case, all the dots on the screen will be bright while the dashes will be dark at all times. So, we get a stationary interference pattern on the screen.

(2) **the waves from S_1 and S_2 differ in phase by π**. In this case, all the dots on the screen will be dark while the dashes will be bright at all times. So, we again get a stationary interference pattern on the screen.

(3) **the waves from S_1 and S_2 differ in any constant phase.** In this case too we get a stationary interference pattern but the points of bright intensity and dark intensity are changed.

(4) **the waves from S_1 and S_2 differ in phase which is changing at random with time.** In this case the interference pattern will also change positions at random, i.e., interference pattern is *not sustained*. We shall observe only an average uniform intensity everywhere on the screen. So, there will be no interference pattern.

In this case, any phase change taking place in one source is simultaneously accompanied by the same change in another source. Therefore, phase change between two sources remains either zero or constant at all times. Thus the two sources derived from a single source act as coherent sources.

The following methods are generally used to produce coherent sources:

1. Young's double slit method.
2. Fresnel's biprism method.
3. Lloyd's single mirror method.

1.4.1 COHERENCE TYPES

Coherence is the basic need for permanent interference pattern. The phase conditions at any point are determined by the following two factors:

(i) Phase condition of two sources, and

(ii) Phase relation developed in their journey towards the superposition point.

Because of the two factors coherence can be expressed with two new terms ‘spatial coherence’ and ‘temporal coherence’.

1. Spatial Coherence

If the phase difference for any two fixed points in a plane normal to the wave propagation does not vary with time, then the wave is said to exhibit spatial coherence.

(i) If the distance between S_1 and S_2 be increased, the fringes appear to lose distinctiveness or contrast.

(ii) If the opening of S be widened, now also the fringes appear to lose sharpness or contrast.

In the above cases, the phase condition of the two sources is disturbed means the waves from them are said to lose spatial coherence and hence no interference is possible.

Condition for spatial coherence

Consider the process of emission of light wave. When an atom falls back from higher energy state (excited state) to lower energy state (unexcited state) a light wave is emitted. This takes a time of the order of 10^{-9} sec. This is called as emission time. Therefore, two atomic emitters may be taken to be correlated in phase for 10^{-9} sec only or less. If it leads the emission time then there will be no coherence between atomic emitters.

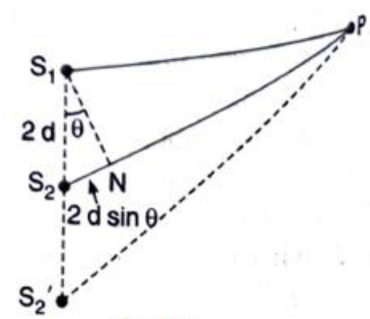


Fig. 1.5

Let S_1 and S_2 be two atomic emitters (sources) and P be a point of observation. The path difference (S_2P - S_1P) = 2 d sinθ. If the path difference (say Δ) is equal or less than $c \times 10^{-9}$, i.e.,

$\Delta = S_2P - S_1P = 2\, d \sin\theta \leq c \times 10^{-9}$

then the two sources are said to maintain spatial coherence. Here, c is velocity of light.

2. Temporal Coherence

Consider the situation in which the path of one beam is extended by introducing a transparent sheet in one path. It is observed that if the path difference exceeds the coherence length $\boldsymbol{l_C = c\, t_C}$, then the interference fringes lose sharpness or contrast. Now we say that the beams lack in temporal coherence. As a result, the fringe pattern disappears.

<u>Note:</u> *To obtain a permanent interference pattern with sufficient contrast, both spatial coherence and temporal coherence are necessary.*

<u>1.5 CONDITIONS FOR INTERFERENCE OF LIGHT</u>

There are certain conditions to obtain permanent or stationary interference pattern. Those are:

(i) Conditions for sustained interference.
(ii) Conditions for observation of the fringes.
(iii) Conditions for good contrast between maxima and minima.

<u>(i) Conditions for sustained interference:</u>

(a) *The two sources should be coherent, i.e., they should vibrate in the same phase or there should be a constant phase difference between them.*

The resultant intensity at any point is given by the expression

$$\boldsymbol{I = (a_1)^2 + (a_2)^2 + 2\, a_1 a_2 \cos\delta}$$

I will be constant if δ remains constant. The total phase difference $\delta = \delta_1 + \delta_2$, where δ_1 is the initial phase difference between the two sources and δ_2 is the phase difference at that point due to path difference between two waves.

(b) *The two sources must emit continuous waves of the same wavelength and time period.*

If the sources do not emit light of the same period, the intensity at any point will be alternately maximum and minimum and hence the interference will not be sustained

(ii) Conditions for observation:

(a) The separation between the two sources (2d) should be small.

When the separation between the two sources is small, the fringe width (λD/2d) is large, so the fringes are separately visible. If 2d is large, fringe width will be small and due to the limited resolving power of eye, the fringes will not be separately visible.

(b) *The distance D between the two sources and screen should be large.*

When D is large, fringe width is large and hence they are separately visible.

(c) *The background should be dark.*

(iii) Conditions for good contrast:

(a) The amplitudes of the interfering waves should be equal or nearly equal

In case of two interfering waves of amplitudes a_1 and a_2 the intensities of maxima and minima are $(a_1 + a_2)$ and $(a_1 - a_2)$ respectively. If the two amplitudes are equal, the minimum intensity would be zero and there will be a good contrast between maxima and minima. If there is a large difference between a_1 and a_2, the intensity of minima will not much differ from that of maxima and the contrast will be poor.

(b) The sources must be narrow, i.e., they must be extremely small.

If the sources are not narrow, the interference between waves from different parts of the same sources will take place and contrast will be poor.

(c) The sources should be monochromatic.

If instead of a monochromatic source, a white light source (different wavelengths) is used, then each separate colour (wavelength) will produce its own interference fringes with its own spacing. The different sets of fringes due to different colours will overlap each other and the net effect would be white light. Only a number of limited fringes can be observed in this case. Hence, the sources should be monochromatic.

1.6 TYPES OF INTERFERENCE

The phenomenon of interference is divided into two classes, namely

(1) division of wavefront and

(2) division of amplitude

(1) Division of wavefront

The incident wavefront is divided into two parts by utilizing the phenomena of reflection, refracting or diffraction. These two parts of the same wavefront travel unequal distances and reunite at some angle to produce interference bands. The Fresnel biprism, Lloyd's mirror, etc. are the examples of this class.

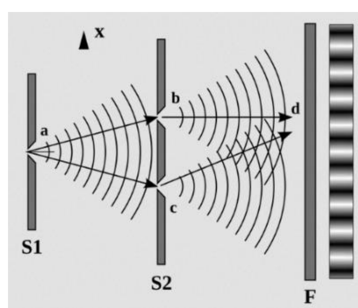


Fig. 1.6

(2) Division of amplitude

The amplitude of the incoming beam is divided into two parts either by parallel reflection or refraction. These divided parts reunite after traversing different paths and produce interference. In this case it is not essential to employ a point or a narrow line source but a broad light source may be employed to produce brighter bands. Newton's rings, Michelson's interferometer, etc. come under this class.

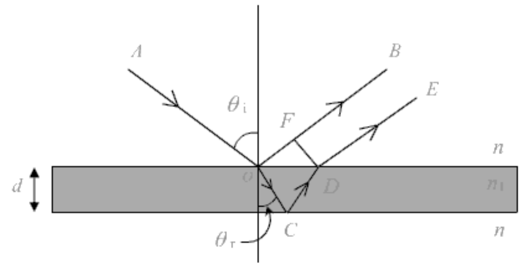


Fig. 1.7

1.7 FRESNEL'S BIPRISM

Biprism

A biprism is essentially combination of two acute prisms placed base to base. The obtuse angle of the prism is about 179^o and the other angles are about 30'each. The action of the biprism is to produce two coherent images of a given slit which are separated at a certain distance and behave as two coherent sources. Fresnel used a biprism to show the phenomenon of interference.

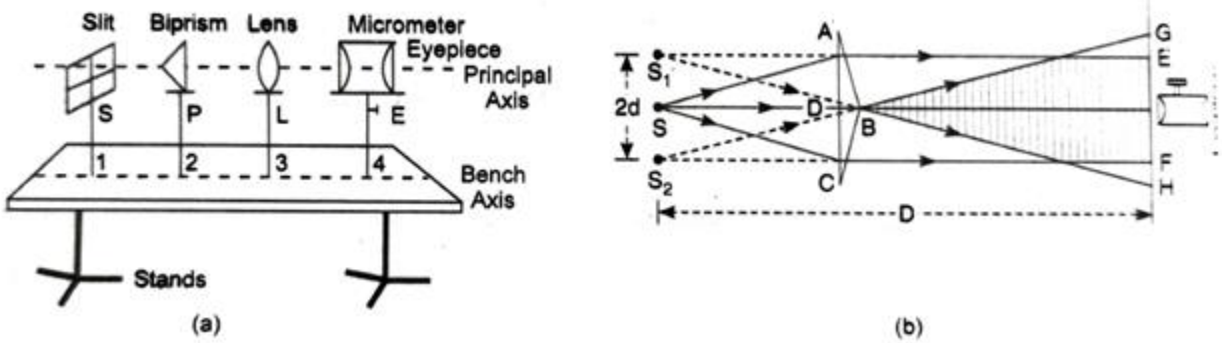


Fig. 1.8

1. Experimental arrangement

The experimental arrangement consists of an optical bench carrying four stands, the first for an adjustable slit, second for biprism, third for lens and the fourth for a micrometer eyepiece. These stands are adjusted at the same height. The stands can be moved along as well as at right angle to the length of optical bench. The slit and biprism can be rotated in their own planes. The slit S is illuminated by a monochromatic source of light such as sodium lamp.

The light emerging from the slit falls on the biprism. The edge B of the biprism divides the incident wave-front into two parts. Firstly, the one which is passing through the upper half ABD of the biprism which deviated through a small angle towards the lower half of the diagram and appears to diverge from virtual image S_1. Secondly, the one which is passing through the lower half CBD is deviated towards the upper half and appears to diverge from the virtual image S_2. The two emergent wavefronts are derived from same wavefront, hence the fundamental conditions of interference is satisfied. S_1 and S_2 serve as coherent sources. The interference fringes are obtained in the overlapping region EF and can be seen by eyepiece.

2. Determination of wavelength of light

The wavelength λ of the light can be determined by the following formula

$$\lambda = \frac{\beta \cdot 2d}{D},$$

where β = fringe width, i.e., spacing between successive maxima or minima,

2d = distance between the two virtual sources.

D = distance between the slit and eyepiece where the fringes are observed.

3. Procedure

In order to determine the wavelength of light the following measurements are made:

(1) Measurement of fringe width β: The vertical cross-wire is adjusted on any successive bright fringes and the readings on micrometer screw are noted. Thus, the mean distance between the two bright fringes β is found out.

(2) Measurement of D: Distance between slit and eyepiece can be read directly on the optical bench. This distance is then corrected from bench error which gives D.

(3) Measurement of 2d: To determine 2d, a lens with focal length less than one-fourth of the distance between biprism and eyepiece is mounted.

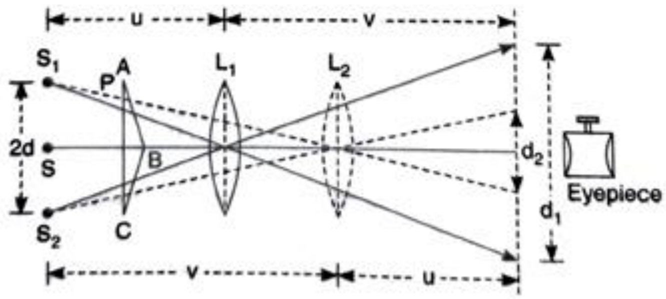


Fig. 1.9

The lens is moved between biprism and eyepiece and adjusted in position L_1 to obtain the sharp images of S_1 and S_2 in eyepiece. The distance d_1 between the real image is noted. Further, the lens is again moved to obtain the sharp images of S_1 and S_2 in eyepiece. Let the position of lens be L_2. The distance d_2 between the real images of S_1 and S_2 is noted by eyepiece.

From the figure,

$$\frac{d_1}{2d} = \frac{v}{u} \text{ and } \frac{d_2}{2d} = \frac{u}{v}$$

$$2d = \sqrt{d_1 d_2}$$

Thus, wavelength λ can be determined by measuring the values β, D and 2d.

SOME FACTS ABOUT INTERFERENCE FRINGESs

We know that fringe-width β is given by $\beta = \lambda D/2d$, where the symbols have their usual meanings.

(1) When the monochromatic source of light is moved nearer to the slit, the fringe-width β does not change, of course, the intensity may increase.

(2) The fringe-width is directly proportional to the wavelength of light used

(3) If the separation between the two coherent sources (2d) is decreased, the fringe-width increases.

(4) As the slit is widened, the fringes disappear gradually because the slit now becomes equivalent to a large number of narrow parallel slits. These slits produce their own fringe system at different places. Due to their overlapping, uniform illumination results.

(5) If white light is used, the central fringe is white. There are few coloured fringes on both sides of the central fringe.

1.8 DETERMINATION OF THE THICKNESS OF A THIN SHEET OF TRANSPARENT MATERIAL

1. White light fringes

We know that white light is a mixture of so many wavelengths varying from 4400 Á (violet) to 7500 A (red). So, the coherent sources of monochromatic light are now equivalent to an infinite number of monochromatic sources of different wavelengths.

According to the formula β, each pair of monochromatic sources of white light produces its own system of fringes with different fringe widths.

The resultant pattern consists of:

(i) Zero order band which is white.

(ii) On both sides of zero order band, few coloured fringes are observed. In coloured fringes, the inner ends are reddish while outer edges are violet.

(iii) After coloured fringes, there is uniform illumination both sides.

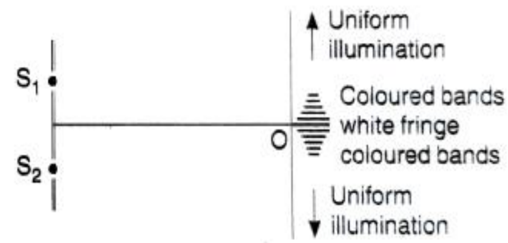


Fig. 1.10

2. Location of zero order fringe

When a monochromatic source of light is used all the bright fringes are of same colour. Hence, it is impossible to locate the zero order fringe. In order to locate the zero order fringe, first of all white light fringes are obtained and the cross-wire is fixed on the zero order fringe (which is white in this case while others are coloured). Now white light source is replaced by the monochromatic source. The fringe now coinciding with cross-wire will be zero order fringe (central fringe) in monochromatic light system.

1.9 STOKE'S PRINCIPLE (CHANGE OF PHASE BY REFLECTION)

According to Stokes, when a light wave is reflected at the surface of an optically denser medium, it suffers a phase change of π, i.e., a path difference of λ/2. It should be remembered that no such phase change is introduced if the reflection takes place from the surface of rarer medium.

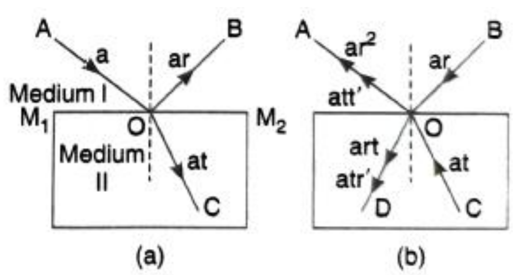


Fig. 1.11

Let us consider a wave AO of light (amplitude *a)* incident at point O on the boundary M_1M_2 of the media 1 and 2. Medium 2 is optically denser than medium 1. The wave is partly reflected along OB (amplitude $a \times r$) and partly transmitted along OC (amplitude $a \times t$). Here r, t be the reflection and transmission coefficients.

Now suppose that the directions of reflected and transmitted wave rays are reversed. On reversing OB, we get a reflected wave OA of amplitude ar^2 ($ar \times r$) and a transmitted wave OD of amplitude art ($ar \times t$). Let r' and t' be the fractions of amplitude reflected and transmitted when the ray is travelling from denser to rarer medium. Now on reversing CO, we get a reflected wave OD of amplitude atr' ($at \times r'$) and transmitted wave OA of amplitude att' ($at \times t'$). As there was no wave originally along OD, hence OD should be zero, i.e.,

$$art + atr' = 0$$

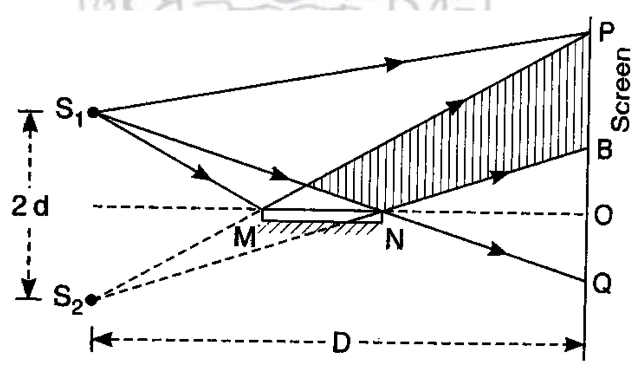
or $$r = -r'$$

The negative sign indicates a phase change of r either at reflection from rarer to denser medium or at reflection from denser to a rarer medium. Due to this reason, In Lloyd's mirror experiment, the central fringe is dark instead of bright because of the interference takes place between direct waves from the source and reflected waves from optically denser medium (glass).

1.10 LIOYD'S SINGLE MIRROR EXPERIMENT

In 1834, Lloyd devised a convenient method to obtain two coherent sources of light to produce sustained interference. The Lloyd's method proved the phase change of π when the reflection takes place at the surface of an optically denser medium.

The experimental arrangement of Lloyd's mirror consists of a plane mirror MN polished on the front surface and blackened at the back to avoid multiple reflections. Light from a narrow slit S_1 illuminated by a monochromatic source of light, is allowed to incident on mirror at grazing angle. The reflected beam appears to diverge from S_2, which is virtual image of S_1. Thus, S_2 and S_1 act as two coherent sources. The direct cone of light PS_1Q and reflected cone of light PS_2B superimpose over each other and produces interference fringes in overlapping region PB of the screen.



1. Zero order fringe

The central zero order fringe is expected to lie at O and it should be bright. But the point O is outside the region of interference and hence the central fringe can’t be seen. However, the central fringe can be seen by moving the screen towards the mirror MN such that the point O of the screen coincides with end N of the mirror. Now the central fringe is formed at O but it is dark. The reason is that the reflected light has suffered a phase charge of π because the direct light cannot undergo any phase change. This clearly indicates that a light beam after reflection from an optically denser medium undergoes a phase change of π.

By illuminating the slit with white light and introducing a thin film of transparent material the entire fringe pattern (including the central fringe) is displaced in the upward direction. So, the entire pattern may be seen. The central fringe is dark and coloured fringes formed on either side.

Mathematical Theory

Here, S_1 is the real source while S_2 is the reflected virtual image of the source S_1. In this way, S_1 and S_2 act as coherent sources producing interference. Here, an additional phase difference is introduced due to reflection. So, we have

$$Y_1 = a\, e^{i(\omega t-\phi_1)}\,\mathbf{j} \text{ and } Y_2 = a\, e^{i(\omega t-\phi_2)}\,\mathbf{j}$$

where $\phi_1 = \frac{2\pi}{\lambda}x_1$ and $\phi_2 = \left(\frac{2\pi}{\lambda}x_2 \pm \pi\right)$

The resultant displacement is given by

$$Y = Y_1 + Y_2 = a\, e^{i(\omega t-\phi_1)}\,\mathbf{j} + a\, e^{i(\omega t-\phi_2)}\,\mathbf{j}$$

$$= a\, e^{i\omega t}\,[e^{-i\phi_1} + e^{-i\phi_2}]\mathbf{j}$$

$$= a\, e^{i\omega t}\,[\cos\phi_1 - i\sin\phi_1 + \cos\phi_2 - i\sin\phi_2]\mathbf{j}$$

$$= a\, e^{i\omega t}\,[(\cos\phi_1 + \cos\phi_2) - i\,(\sin\phi_1 + \sin\phi_2)]\mathbf{j}$$

$$= e^{i\omega t}\,[a(\cos\phi_1 + \cos\phi_2) - i\,a(\sin\phi_1 + \sin\phi_2)]\mathbf{j}$$

$$= R\, e^{i\omega t}\, e^{-i\theta}\mathrm{j} = R\, e^{i(\omega t-\theta)}\,\mathrm{j} \qquad (2)$$

Now $$y = R\, e^{i(\omega t-\theta)}\,\mathrm{j}\cdot\mathrm{j} = R\, e^{i(\omega t-\theta)} \qquad (3)$$

Here $$R\cos\theta = a(\cos\phi_1 + \cos\phi_2) \qquad (4)$$

$$R\sin\theta = a(\sin\phi_1 + \sin\phi_2) \qquad (5)$$

$$\therefore I = R^2 = a^2[\cos^2\phi_1 + \cos^2\phi_2 + 2\cos\phi_1\cos\phi_2 + \sin^2\phi_1 + \sin^2\phi_2 + 2\sin\phi_1\sin\phi_2]$$

$$= 2a^2[1 + \cos\,(\phi_2 - \phi_1)]$$

$$= 4a^2\cos^2\frac{(\phi_2-\phi_1)}{2} \qquad (6)$$

(1) Intensity is maximum, when

$$\cos\frac{(\phi_2 - \phi_1)}{2} = 1$$

$$\phi_2 - \phi_1 = 2n\pi$$

Or $$\frac{2\pi}{\lambda}x_2 - \frac{2\pi}{\lambda}x_1 \pm \pi = 2n\pi$$

$$\frac{2\pi}{\lambda}(x_2 - x_1) \pm \pi = 2n\pi$$

$$\frac{2\pi}{\lambda}(x_2 - x_1) = (2n \pm 1)\,\pi$$

$$(x_2 - x_1) = (2n \pm 1)\frac{\lambda}{2} \qquad (7)$$

So, the maximum intensity occurs when the path difference is an odd multiple of λ/2.

(2) Intensity is minimum when

$$\cos\frac{(\phi_2 - \phi_1)}{2} = 0$$

$$(\phi_2 - \phi_1)/2 = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \phi_2 - \phi_1 = (2n+1)\pi$$

$$\frac{2\pi}{\lambda}(x_2 - x_1) = (2n+1)\pi \pm \pi = (2n+2)\pi \;or\; 2n\pi$$

$$(x_2 - x_1) = (2n+2)\frac{\lambda}{2} \;or\; n\lambda \qquad (8)$$

Thus, when path difference is an even multiple of λ/2, the intensity becomes zero.

Thus, when $x_1 = x_2$, we get the central fringe. This satisfies the condition of minima.

3. Determination of wavelength

For the determination of wavelength, Lloyd's mirror is mounted vertically on an upright of an optical bench and placed along the length of the bench. On another upright, a narrow vertical slit is mounted and is illuminated by a monochromatic source of light. Now the mirror is rotated about an axis parallel to the length of bench until distinct fringes are observed in a micrometer eyepiece. The plane of the mirror is now exactly parallel to the length of the slit.

As in case of biprism experiment, the fringe width B is measured by micrometer eyepiece. The distance 2d between the two virtual sources is measured by displacement method using a convex lens. The distance D between screen and slit is measured by metre scale. Now the wavelength of monochromatic source of light can be calculated by using the following formula:

$$\beta = \frac{\lambda D}{2d} or \lambda = \beta . \frac{2d}{D}$$

<u>1.11 INTERFERENCE BY A PLANE PARALLEL FILM ILLUMINATED BY A PLANE WAVE</u>

Let a plane wavefront be allowed to incident normally on a thin film of uniform thickness t. The plane wavefront is obtained with the help of a partially reflecting glass plate G inclined at an angle 45° with the parallel monochromatic beam of light. The plane wavefront is partly reflected at the upper surface of the film and partly transmitted into the film. The transmitted wavefront is reflected again Monochromatic A radiation from the bottom surface of the film and emerges through the first surface. The wavefront reflected from the upper surface and the lower surface interfere with each other. The resulting interference pattern can be observed with eye E without obstructing the incident wavefront.

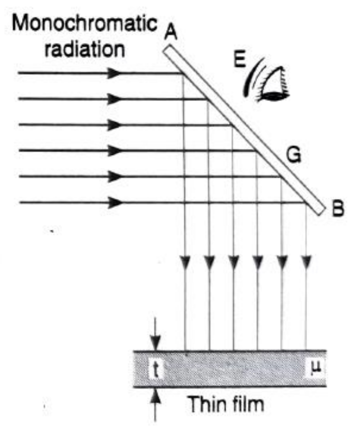


Fig. 1.13

Here, we consider the following two points:

(i) The wave reflected from the lower surface of the film traverses an additional path 2u t [ut from upper surface to lower surface and u t from lower surface to upper surface where is the refractive index of the film.

(ii) When the film is placed in air, the wavefront reflected from the upper surface undergoes an additional phase change of t (because the reflection takes place at the surface of a denser medium). Here, it should be noted that no phase change takes place at lower surface because the reflection takes place at the surface of rarer medium. Thus, the conditions of maxima and minima are changed.

$2\mu t = n\lambda$ conditions of destructive interference

$2\mu t = (2n+1)\lambda/2$ conditions of constructive interference

where n = 0, 1,2, 3, …

Thus, the film appears bright when the path difference $2\mu t = (2n+1)\lambda/2$ and

appears dark when the path difference $2\mu t = n\lambda$.

1.12 OBLIQUE INCIDENCE OF A PLANE WAVE ON A THIN FILM (THE COSINE LAW)

Let GH and G_1H_1 be the two surfaces of a transparent film of uniform thickness t and refractive index μ as shown in figure. Suppose a monochromatic light ray AB be incident on its upper surface. This ray is partly reflected along BR and refracted along BC. After one internal reflection at C, we obtain the ray CD. After refraction at D, the ray finally emerges out along DR_1 in air.

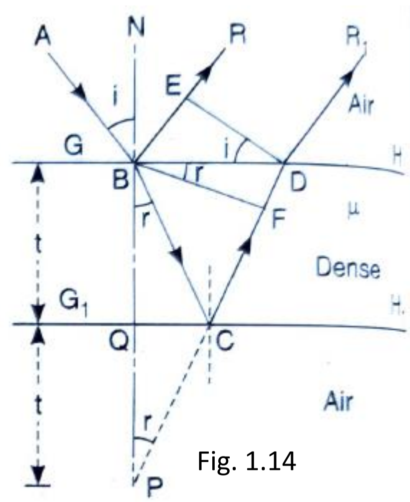


Fig. 1.14

Obviously, DR_1 is parallel to BR. To find the effective path difference between the rays BR and DR_1, we draw a normal DE on BR and normal BF on DC. We also extend DC in the backward direction and BQ meets at P. In the figure the angle of incidence ∠ABN = i, and the angle of refraction ∠QBC = r. From the figure ∠BDE = i , ∠FBD = r and ∠QPC = r.

The optical path difference between the two reflected light rays (BR and DR_1) is given by

$$\Delta = \text{Path (BC + CD) in film - Path BE in air}$$
$$= \mu\,(BC + CD) - BE \qquad (1)$$

We know that $\mu = \dfrac{\sin i}{\sin r} = \dfrac{\left(\dfrac{BE}{BD}\right)}{\left(\dfrac{FD}{BD}\right)} = \dfrac{BE}{FD}$ [from triangles BED & BFD]

$$BE = \mu\,(FD) \qquad (2)$$

From eqs. (1) and (2),

$$\Delta = \mu\,(BC + CD) - \mu\,(FD)$$
$$= \mu\,(BC + CF + FD) - \mu\,(FD)$$
$$= \mu\,(BC + CF)$$
$$= \mu\,(PF) \qquad (3) \qquad (\because BC = PC)$$

From triangle BPF, cos r = PF/BP

PF= BP cos r = 2t cos r (4)

Substituting the PF value in eq. (3), we have

$$\text{Optical Path difference } \Delta = \mu \times 2t\cos r = 2\mu t\cos r \qquad (5)$$

This is called cosine law.

A ray reflected at a surface backed by a denser medium suffers an abrupt phase change of π is equivalent to a path difference λ/2.

Thus, the effective path difference between the two reflected rays is (2μt cos r + λ/2).

We know that maxima occurs when effective path difference Δ = nλ.

$$2\mu t \cos r \pm \lambda/2 = n\lambda$$
$$2\mu t \cos r = (2n \pm 1)\,\lambda/2 \qquad \text{------ (A)}$$

If this condition is fulfilled, the film will appear bright in the reflected light.

The minima occurs when the effective path difference is (2n ± 1) λ/2, i.e.,

$$2\mu t \cos r \pm \lambda/2 = (2n \pm 1)\,\lambda/2$$

$$2\mu t \cos r = (2n \pm 1)\,\lambda/2 \pm \lambda/2$$

$$2\mu t \cos r = n\lambda \qquad \text{------ (B)}$$

because (n +1) or (n -1) can also be taken as integer, when n is integer. Here, n =0,1,2,3,...etc. When this condition is fulfilled, the film will appear dark in the reflected light

Alternative Treatment of Cosine Law

Let GH and G_1H_1 be the two surfaces of a transparent film of uniform thickness t and effective index μ as shown in figure. Suppose a ray AB of monochromatic light is incident on its upper surface. This ray is partly reflected along BR and refracted along BC. After one reflection at C, we obtain the ray CD. After refraction at D, the ray finally emerges out along DR_1 in air. Obviously, DR_1 is parallel to BR. Our aim is to find out the effective path difference between the rays BR and DR_1. For this purpose, we draw a normal DE on BR.

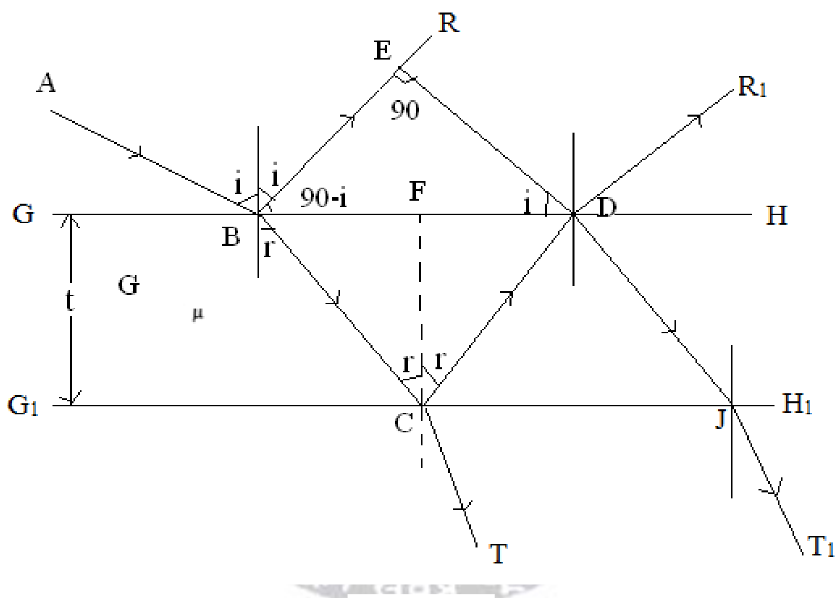


Fig. 1.15

The path difference Δ is given by

$$\Delta = \mu\,(BC+CD) - BE \qquad ...(1)$$

From triangle BCF, cos r = CF/BC or t/BC = cos r

$$BC = CD = t/\cos r \qquad ...(2)$$

In order to calculate BE, we first find BD which is equal to (BF + FD).

We consider triangle BFC,

$$\tan r = BF/FC \quad \text{or} \quad \tan r = BF/t$$

$$BF = t \tan r$$

$$BD = BF + FD = 2\,BF = 2t \tan r$$

From triangle BED, sin i = BE/BD

⇨ $BE = BD \sin i = 2t \tan r \sin i$

We know that

$$\frac{\sin i}{\sin r} = \mu \; or \sin i = \mu \sin r$$

$$BE = 2t \tan r\,(\mu \sin r)$$

or BE =2μt tan r sin r

From eqs. (2) and (3), substituting the values of BC and BE in eq. (1), we get

$$\Delta = \mu(\frac{2t}{\cos r}) - 2\mu t \tan r sin r$$

$$= \frac{2\mu t}{\cos r} - 2\mu t \frac{sin^2 r}{cos r}$$

$$= \frac{2\mu t}{\cos r}[1 - sin^2 r]$$

$$= \frac{2\mu t}{\cos r} cos^2 r$$

$$\Delta = 2\mu t \cos r$$

This is known as cosine law.

Here the ray reflected at a surface backed by a denser medium suffer an abrupt phase change of π (≈ path difference λ/2). Thus, the effective path difference between the two reflected rays is 2μt cosr ± λ/2.

We know that maxima occurs when effective path difference Δ = nλ.

i.e., 2 μt cos r ± λ/2 = nλ

2 μt cos r = (2n ± 1) λ/2

If path difference meets the above condition, the film will appear bright in the reflected light.

The minima occurs when the effective path difference is (2n ± 1) λ/2,

i.e., 2 μt cos r ± λ/2 = (2n ± 1) λ/2

2 μt cos r ± λ/2 = 2n λ/2 ± λ/2

2 μt cos r = nλ, where, n =0,1,2,3,...etc.

If path difference meets the above condition, the film will appear dark in the reflected light.

1.12.1 INTERFERENCE DUE TO TRANSMITTED LIGHT (COSINE LAW)

Let GH and G_1H_1 be the two surfaces of a transparent film of uniform thickness t and effective index μ as shown in figure. Suppose a ray AB of monochromatic light is incident on its upper surface. Due to simultaneous reflection and refraction, we obtain two transmitted rays CT and ET_1. These rays have originated from the same point source (have constant phase difference) and are able to produce sustained interference when combined. In order to calculate the path difference between the two transmitted rays, we draw normal CQ and EP on DE and CT respectively. We also produce ED in the backward direction which meets produced CF at I.

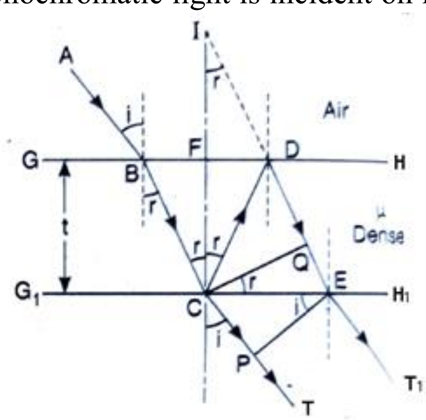


Fig. 1.16

The effective path difference is given by

Δ =μ (CD +DE) – CP …(1)

From triangle CEP, sin i = CP/CE

From triangle QCE sin r = QE/CE

We know that, $\frac{sin\ i}{sin\ r} = \mu$

$$\frac{(CP\ /CE)}{(\mathrm{QE}\ /\ \mathrm{CE})} = \mu$$

$$\frac{CP}{QE} = \mu \ or\ CP\ = \mu \qquad \ldots (2)$$

From eqs. (1) and (2),

$$\Delta = \mu\ (\mathrm{CD} + \mathrm{DQ} + \mathrm{QE}) - \mu\ (\mathrm{QE})$$

$$= \mu(\mathrm{CD} + \mathrm{DQ}) = \mu\ (\mathrm{QI}) \quad (\because \mathrm{CD} = \mathrm{ID})$$

$$\Delta = 2\mu\ t \cos r$$

Here, inside the film, reflection backed by rarer medium (air) so, no abrupt change of π takes place.

The maxima occurs when effective path difference $\Delta = n\lambda$

$$2\ \mu t \cos r = n\lambda$$

This is the condition for bright fringe in film in transmitted light.

The minima occurs when the effective path difference is $(2n \pm 1)\ \lambda/2$, i.e.,

$$2\ \mu t \cos r = (2n \pm 1)\ \lambda/2, \text{ where } n=0,1,2,3, ..., \text{etc.}$$

This is the condition for dark fringe in film in transmitted light.

Note: The conditions of maxima and minima in transmitted light are just reverse of the conditions reflected light.

1.13 COLOURS OF THIN FILMS

When a thin film is exposed to a white light source beautiful colours are observed. Suppose incident light is white and falls on a parallel sided film. The incident light will split up by reflection at the top and bottom of the film. The splitted rays are in a position to interfere and the interference of these rays is responsible for the colours. The bright or dark appearance of the reflected light depends upon μ, i and r. In case of white light, even r made constant, μ varies with wavelength.

Fig. 1.17

At a particular point of the film and for a particular position of the eye, only certain wavelengths of the interfering rays will satisfy the conditions of bright fringe ($\Delta = n\lambda$). Hence, only such wavelengths (colours) will be present there. Other neighbouring wavelengths will be present with reduced intensity. The colours for which the conditions of minima ($\Delta = (2n+1)\lambda/2$) is satisfied are absent.

In a similar way, we can say that if the same point of the film is observed with eye in different positions or different points of the film are observed with eye in the same position, a different set of colours is observed each time.

1.14 INTERFERENCE BY APLANE PARALLEL FILM ILLUMINATED BY A POINT SOURCE

Consider a plane parallel film of uniform thickness t and refractive index μ. Suppose light from a point source S is allowed to fall on the transparent film. The distance of the source S from the upper surface AB is SK. The wave SQ reflected from the upper surface (as shown by 1) appears to emerge from S such that SK = KS_1. Further, the wave reflected from the lower surface (coming in the form of wave 2) will appear to emerge from S_2 such that $KS_2 = KS_1 + S_1S_2 = KS + S_1S_2$.

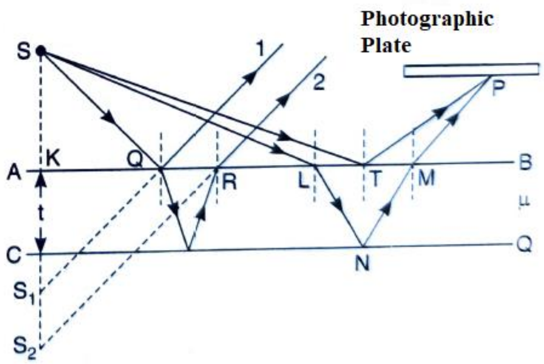


Fig. 1.18

Thus, for very nearly normal incidence, S_1 and S_2 act as two coherent sources and produce interference pattern. The interference pattern will be the same as observed in Young's double slit experiment. The interference pattern can be recorded on a photographic plate. When the photographic plate is parallel to the reflecting surface, dark and bright rings are observed. The path difference between the waves reaching at P is given by

Path difference $\Delta = SL + \mu\,(LN + NM) + MP - (ST + TP)$

The condition for maximum intensities is $\Delta = (2n+1)\frac{\lambda}{2}$

The condition for minimum intensities is $\Delta = n\lambda$

Here, the phase change of π occurs when and only the ray undergoes reflection at a source backed by denser medium.

1.15 INTERFERENCE BY A FILM WITH TWO NON-PARALLEL REFLECTING SURFACES (WEDGE-SHAPED FILM)

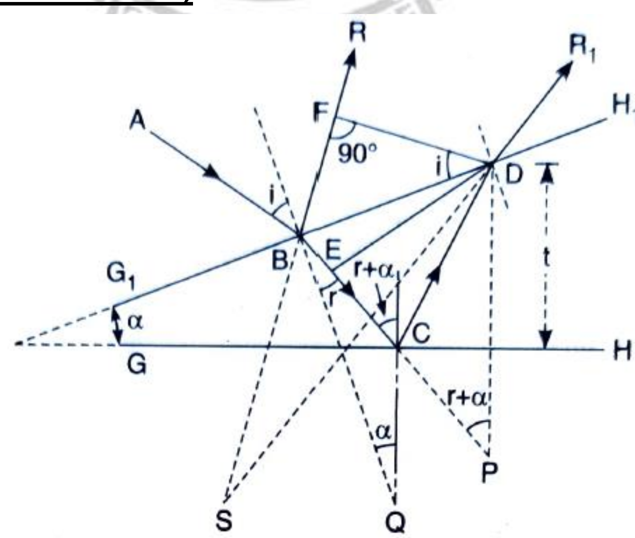


Fig. 1.19

Consider two plane surfaces GH and GH_1 inclined at an angle α and enclosing a wedge shape air film. The thickness of air film increasing from G to H as shown in figure. Let μ be the refractive index of the material of this film. When this film is illuminated by sodium light, then the interference between reflected rays, one reflected from the front surface and the other obtained by internal reflection at the back surface will take place. It is observed from the figure that the interfering waves BR and DR_1 are not parallel but appear to diverge from a point S. Thus, the interference takes place at S which is virtual. In order to consider the interference between these two waves, we will first calculate the path difference between them.

The optical path difference Δ is given by

$$\Delta = \mu\,(BC+CD) - BF$$
$$= \mu\,(BE + EC + CD) - \mu\, BE \qquad (\because BF = \mu\, BE)$$
$$= \mu\,(EC + CD) = \mu\,(EP)$$
$$= 2\mu t \cos(r + \alpha)$$

Due to reflection, an additional phase change π of path difference $\lambda/2$ is introduced.

Hence, the final path difference $\Delta = 2\mu t \cos(r+\alpha) \pm \lambda/2$

For constructive interference, $\Delta = n\,\lambda$

$$2\mu t\, \cos(r+\alpha) \pm \lambda/2 = n\,\lambda$$
$$2\mu t\, \cos(r+\alpha) = (2n\pm 1)\,\lambda/2 \qquad (For\ maxima)$$

For destructive interference, $\Delta = (2n\pm 1)\,\lambda/2$

$$2\mu t\, \cos(r+\alpha) \pm \lambda/2 = (2n\pm 1)\,\lambda/2$$
$$2\mu t\, \cos(r+\alpha) = n\,\lambda \qquad (For\ minima)$$

where n = 0, 1, 2,3, ..., etc.

DETERMINATION OF ANGLE OF WEDGE OR DIAMETER OF A WIRE

Two rectangular pieces of ordinary glass plates are taken. The wire is placed between the two plates at one end and the two plates are tied together at another end to form an air-wedge. The thickness of the wedge gradually increases from one end to the other end. Light from a monochromatic source of light S is allowed to incident normally on the combination by a glass plate G inclined at an angle 45° with the horizontal. A microscope M is focused on the top of the wedge as shown in figure. Light rays reflected from the wedge are brought to a focus at the centre of the cross-wire. The ray reflected at the bottom surface of the top plate and partly from the top surface of the lower plate are in a condition to interfere. Hence, alternate dark and bright fringes of equal thickness like AB, CD, EF, etc. are observed. Let P and Q correspond to the centre of n and $(n+1)^{st}$ dark fringes. Suppose the thickness of the film at these points are t_1 and t_2 respectively.

For air film $\mu = 1$, and normal incidence r = 0, we have

Path difference $2\mu t \cos r$ become 2t

i.e., $$2t_1 = n\lambda \qquad \text{------ (1)}$$
$$2t_2 = (n+1)\lambda \qquad \text{------ (2)}$$

From figure,

$$\tan\alpha = \frac{t_1}{Ap} = \frac{t_2}{AQ} \quad \text{or} \quad \frac{t_2 - t_1}{AQ - AP} = \tan\alpha$$

But Fring width $\beta = AQ - AP$

$$\therefore (t_2 - t_1) = \beta \tan\alpha \qquad \text{------ (3)}$$

From eqs. (1) and (2),

$$(t_2 - t_1) = \frac{\lambda}{2} \qquad \text{------ (4)}$$

From eqs. (3) and (4), we get $\beta \tan\alpha = \frac{\lambda}{2}$

$$\beta = \frac{\lambda}{2\tan\alpha} = \frac{\lambda}{2\,\alpha} \qquad \text{------ (5)}$$

Here, β is independent of n and hence all the dark fringes are equally spaced.

The cross-wire of the microscope is fixed on the n^{th} bright fringe and the reading of microscope is noted. Now, the microscope is moved and the cross-wire is fixed on $(n+1)^{th}$ bright fringe. The reading on microscope is noted. The difference of the two readings gives β. Putting the value of β in eq. (5) we can determine the angle α of the wedge.

If the wire is of diameter d at a distance x from the line of contact of the two plates, then

$$\tan\alpha = \frac{d}{x}$$

But $$\tan\alpha = \frac{\lambda}{2\beta}$$

Now $$\frac{d}{x} = \frac{\lambda}{2\beta} \quad \text{or} \quad d = \frac{\lambda x}{2\beta}$$

1.16 NEWTON'S RINGS

A Plano convex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate. An air film is formed between the lower surface of the lens and the upper surface of the glass plate. The thickness of the air film is zero, at the point of contact O and gradually increases from the point of contact outwards. If monochromatic light is allowed to fall normally on this film a system of alternate bright and dark concentric rings are formed. They are called Newton's rings. Since, the phenomena was first described by Newton that is the rings are known as Newton's rings after his name.

1. Experimental arrangement

The experimental arrangement of obtaining Newton's rings is shown in figure. S is a monochromatic source of light.

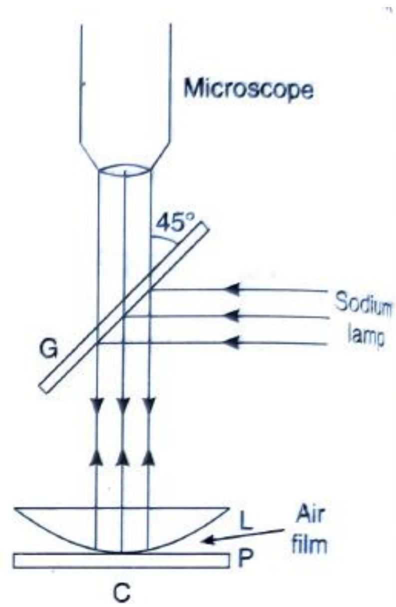


Fig. 1.20

The light from S is rendered parallel by a convex lens L_1. These horizontal parallel rays fall on a glass plate G which is inclined at 45^0 and reflected from it. This reflected beam falls normally on the lens L placed on the glass plate PQ. Interference occurs between the rays reflected from the upper (say ray1) and lower surfaces (say ray2) of the film. The interference rings are viewed through a microscope M.

Explanation of the formation of Newton's rings

Newton's rings are formed due to interference between the waves reflected from the top and bottom surfaces of the air film formed between the plates. AB is a monochromatic ray of light which falls on the system. A part is reflected at C (glass-air boundary) which goes out in the form of rays I without any phase reversal. The other part is refracted along CD. At point D it is again reflected and goes out in the form of ray 2 with a phase reversal of π. The reflected rays 1 and 2 are in a position to produce interference fringes as they have been derived from the same ray AB and hence fulfil the condition of interference. The path difference between them is $2\mu t \cos r + \lambda/2$. For air film and for normal incidence (μ=1, r = 0), the path difference is $(2t + \lambda/2)$.

Theory of Newton's rings

Now we shall calculate the diameters of dark and bright rings. Let LOL' be the lens placed on a glass plate AB. The curved surface LOL' is the part of spherical surface (shown dotted in fig. 13) with centre at C. Let R be the radius of curvature and r be the radius of Newton's ring corresponding to the constant film thickness t. As discussed above, for all integral values of n,

Newton's rings by reflected light

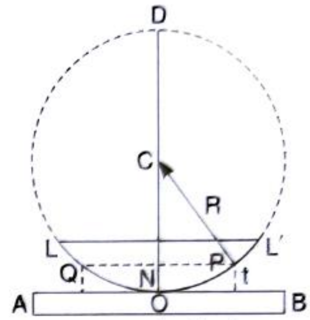


Fig. 1.21

condition for dark ring $2t = n\lambda$ (1)

condition for bright ring $2t = (2n-1)\,\lambda$ (2)

From the figure, by the property of circle

$$NP \times NQ = NO \times ND$$

$$r \times r = t \times (2R - t)$$

$$= 2Rt - t \approx 2Rt$$

$$r^2 = 2Rt$$

$$\text{or} \quad t = \frac{r^2}{2R}$$

For a bright ring,

$$2\frac{r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

But r=D/2, where D is the diameter of the bright ring

$$D^2 = 4(2n\text{-}1)\frac{\lambda R}{2}$$

$$D^2 = 2(2n\text{-}1)\lambda R$$

$$D^2 \propto (2n\text{-}1)$$

$$D \propto \sqrt{2n\text{-}1}$$

Diameter of the bright rings are proportional to the square root of the odd numbers.

For Dark ring,

$$2\frac{r^2}{2R} = n\lambda$$

$$r^2 = n\lambda R$$

$$D^2 = 4n\lambda R$$

$$D = 2\sqrt{\lambda R}$$

$$D \propto \sqrt{n}$$

Thus, diameters of dark rings are proportional to the square roots of natural numbers.

It can be shown that fringe width decreases with the order of the fringe and fringes get closer with increase in their order.

Example: The difference in 1st and 4th ring (~ 3 rings) diameters is equal to the difference between 9th and 16th rings (~7 rings) i.e., $2\sqrt{\lambda R}$

Newton's rings by transmitted light

In case of transmitted light

For bright ring, $2\mu t \cos r = n\lambda$

$\Rightarrow \quad 2t = n\lambda$

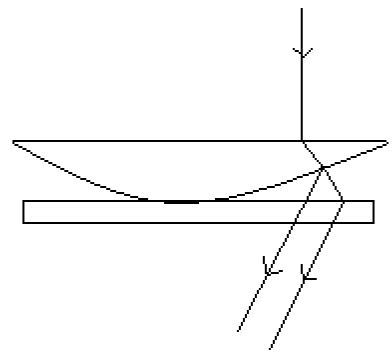


Fig. 1.22

We know that $2t = \dfrac{r^2}{R}$

$$\frac{r^2}{R} = n\lambda$$

$$r^2 = n\lambda R$$

$$D^2 = 4n\lambda R$$

$$D = 2\sqrt{\lambda R}$$

$$D \propto \sqrt{n}$$

i.e., for constructive interference $D\,\alpha\,\sqrt{n}$ which is the condition for destructive interference in reflected light.

For bright ring, $2\mu t \cos r = (2n\text{-}1)\,\lambda$

$\Rightarrow \quad 2t = (2n\text{-}1)\,\lambda$

$\Rightarrow \quad 2\dfrac{r^2}{2R} = (2n-1)\dfrac{\lambda}{2} \quad or \quad r^2 = \dfrac{(2n-1)\lambda R}{2}$

But r =D/2,

$$D^2 = 4(2n\text{-}1)\frac{\lambda R}{2}$$

$$D^2 = 2(2n\text{-}1)\lambda R$$

$$D^2 \propto (2n\text{-}1)$$

$$D \propto \sqrt{2n\text{-}1}$$

i.e., for destructive interference $D \propto \sqrt{2n+1}$ which is the condition for constructive interference in reflected light.

Hence, the rings obtained in transmitted light are just opposite to the rings in reflected light.

1.16.1 DETERMINATION OF WAVELENGTH OF SODIUM LIGHT USING NEWTON'S RINGS

The experimental arrangement is shown in fig. Let R be radius of curvature of the surface in contact with the plate, λ the wavelength of light used and D_n and D_{n+p} the diameters of n^{th} and $(n+p)^{th}$ dark rings respectively, then

$$D_n{}^2 = 4n\lambda R$$

$$\left(D_{n+p}\right)^2 = 4(n+p)\lambda R$$

$$\left(D_{n+p}\right)^2 - D_n{}^2 = 4p\lambda R$$

$$\lambda = \frac{\left(D_{n+p}\right)^2 - D_n^{\ 2}}{4pR} \qquad (1)$$

Using this formula, λ can be determined.

Procedure:

i. First of all the eyepiece of the microscope is adjusted on its cross-wires.
ii. Now, the distance of the microscope from the film is adjusted at the rings with dark centre in well focus.
iii. The centre of the cross-wires is adjusted at the centre of the fringe pattern.
iv. By counting the number of fringes, the microscope is moved to the extreme left of the pattern and the cross-wire is adjusted tangentially in the middle of a clearly n^{th} (say 20^{th}) bright or dark fringe. The reading of micrometer position is noted.
v. The microscope is now moved to the right and the readings of micrometer screw are noted successively at $(n-2)^{th}$ (say 18^{th}), $(n-4)^{th}$ (say 16^{th})...... rings etc. till we are very near to the central dark spot.
vi. Again crossing the central dark spot in the same direction (right side) the readings corresponding to … $(n-4)^{th}$ (16^{th}), $(n-2)^{th}$ (18^{th}), (20^{th}) rings are noted on other side.
vii. Now, a graph is plotted between number of rings n and the square of the corresponding diameter.

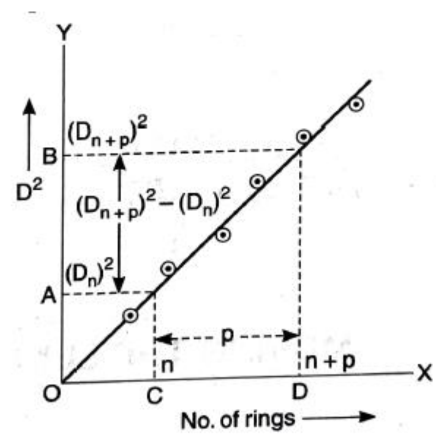


Fig. 1.23

From the graph

$$\frac{\left(D_{n+p}\right)^2 - D_n^{\ 2}}{p} = \frac{AB}{CD}$$

The radius R of the plano-convex lens can be obtained with the help of spherometer using the following formula.

$$R = \frac{l^2}{6h} + \frac{h}{2}$$

Where *l* is the distance between the two legs of the spherometer and h is the difference of the readings of the spherometer when it is placed on the lens as well as when placed on plane surface.

<u>DETERMINATION OF REFRACTIVE INDEX OF A LIQUID</u>

With the help of travelling microscope the diameters of number of dark rings are measured in the following manner.

i. The position of the microscope is adjusted such that the center of Newton's rings at the point of intersection of cross wires.
ii. The microscope is moved until the vertical cross wire is tangential to the 16^{th} dark ring. The microscope reading is noted.
iii. Then the microscope is moved such that the cross wire is successively tangential to 12^{th}, 8^{th}, 4^{th} dark rings. The readings are noted in each case.
iv. Reading corresponding to the same rings are taken on the other side of the center.
v. The diameter of n^{th} and $n+p^{th}$ rings with air is given by

$$D_n^2 = 4n\lambda R$$

$$\left(D_{n+p}\right)^2 = 4\left(n+p\right)\lambda R$$

$$\left(D_{n+p}\right)^2 - D_n^2 = 4p\lambda R \qquad (1)$$

Now, the whole arrangement is placed in a container and liquid is poured in the container whose refractive index is to be determined.

Again the diameters of n^{th} ring and $(n + p)^{th}$ ring are determined. So there is a liquid film between glass plate and plano-convex lens.

Now we have

$$D'^2_n = \frac{4n\lambda R}{\mu}$$

$$(D'_{n+p})^2 = \frac{4(n+p)\lambda R}{\mu}$$

$$\left(D'_{n+p}\right)^2 - D'^2_n = \frac{4p\lambda R}{\mu} \qquad (2)$$

Dividing eq. (1) by (2), we get

$$\mu = \frac{\left(D_{n+p}\right)^2 - D_n^2}{\left(D'_{n+p}\right)^2 - D'^2_n} \qquad (3)$$

Using this formula we can calculate the refractive index of the liquid.

1.17 MICHELSON INTERFEROMETER

Construction

The apparatus is shown in figure. It consists of two excellent optically plane, highly polished mirrors M_1 and M_2 which are at right angles to each other. There are two optically flat glass plates G_1 and G_2 of same thickness and of the same material placed parallel to each other. These plates are also inclined at an angle 45° with the mirrors M_1 and M_2. The face of G_1 towards G_2 is semi-silvered.

The mirror M_1 is mounted on a carriage which can be moved forward or backward. The motion is controlled by a very fine micrometer screw. Fine mirrors M_1 and M_2 are provided with three levelling screws at their backs. With the help of these screws the mirror can be tilted about horizontal and vertical axes so that they can be made exactly perpendicular to each other, T is telescope which receives the reflected lights from mirrors M_1 and M_2.

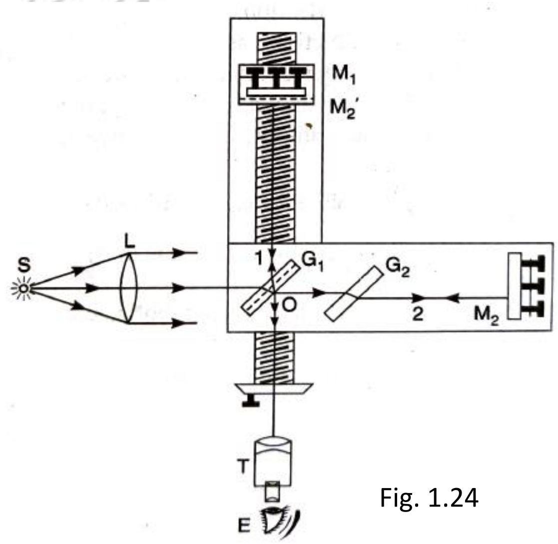


Fig. 1.24

Working

Light from the monochromatic source S is rendered parallel by using a lens L and falls on the glass plate G_1 at an angle of 45^0. At the back surface of G_1 it is partly reflected along ray1 and remaining part transmitted along ray 2.

The reflected beam move towards M_1 and falls normally on it. It is reflected back along the same path and emerges along OT. The transmitted beam AB falls normally on M_2. It is reflected along the same path and after reflection at the back side of G_1, it travels along OT. The two emergent beams have been derived from a single incident beam. Hence they are coherent. Thus the two beams produce interference fringes in the field of view of the telescope.

A desired path difference can be introduced between the two reflected rays by moving the mirror M_1. It can be noted from the figure that ray 1 passes through G_1 twice whereas the ray No. 2 does not do so even once. Thus, in the absence of glass plate G_2, the two paths OM_1 and OM_2 are not equal. To equalize the two paths, a glass plate G_2 of same thickness and material as that of G_1 is introduced in the path of ray 2. So the glass plate G_2 is called as compensating plate.

Looking in the direction M_1 from E, one observes M_1 and also a virtual image M_2' of M_2 formed in G_1. Thus, the two interfering beams come by reflection from mirror M_1 and the other which is reflected from M_2'.

Hence, the Michelson interferometer is optically equivalent to an air film between M_1 and M_2'. If the distance $OM_1 = x_1$ and $OM_2' = x_2$, then
the path difference between interfering rays is given by

$$\Delta = 2(x_1 \sim x_2) \qquad (\textit{When glass plate is heavily polished})$$

$$\Delta = 2(x_1 \sim x_2) \pm \lambda/2 \qquad (\textit{When silvereing of glass plate is thin})$$

The interference fringes may be straight, circular, parabolic, etc. depending upon path difference and the angle between mirrors M_1 and M_2.

Mathematical Analysis

At the back of glass plate G_1 the beam is divided into two parts: one going to mirror M_1, and reflected by this mirror, at a distance x_1 and other reflected by mirror M_2' (mirror image at M_2) at distance x_2. The respective displacements of a particle at O due to component waves are expressed as

$$y_1 = a \exp.\left[i\left\{\omega t - k\left(2x_1 + \frac{\lambda}{2}\right)\right\}\right] \qquad \textbf{(1)}$$

$$y_2 = a \exp.\left[i\left\{\omega t - k\left(2x_2 + \frac{\lambda}{2}\right)\right\}\right]. \qquad \textbf{(2)}$$

The factor $\frac{\lambda}{2}$ in the phase represents a phase change of $\boldsymbol{\pi}$ due to reflection at the mirror.

The resultant displacement is given by

$$\begin{aligned} y &= y_1 + y_2 \\ &= a \exp.\left[i\left\{\omega t - k\left(2x_1 + \frac{\lambda}{2}\right)\right\}\right] + a \exp.\left[i\left\{\omega t - k\left(2x_2 + \frac{\lambda}{2}\right)\right\}\right] \\ &= a \exp.(i\omega t).\left[\exp\left\{-ik\left(2x_1 + \frac{\lambda}{2}\right)\right\} + \exp.\left\{-ik\left(2x_2 + \frac{\lambda}{2}\right)\right\}\right] \qquad \ldots(3) \end{aligned}$$

Let $$k\left(2x_1 + \frac{\lambda}{2}\right) = \alpha_1 \text{ and } k\left(2x_2 + \frac{\lambda}{2}\right) = (\alpha_1 - \theta)$$

or $$\theta = k\left(2x_1 + \frac{\lambda}{2}\right) - k\left(2x_2 + \frac{\lambda}{2}\right) = 2k(x_1 - x_2) \qquad \ldots(4)$$

Substituting these values in eq. (3), we get

$$y = a \exp.(i\omega t)[\exp.(-i\alpha_1) + \exp.\{-i(\alpha_i - \theta)\}]$$
$$= a \exp.(i\omega t).\exp.(-i\alpha_1)[1 + \exp.(i\theta)]$$
$$= a \exp.\{i(\omega t - \alpha_1)\}\left[\exp.\left(-\frac{i\theta}{2}\right) + \exp.\left(\frac{i\theta}{2}\right)\right]\exp.\left(\frac{i\theta}{2}\right)$$
$$= 2a \exp.\{i(\omega t - \alpha_1)\}\cos\frac{\theta}{2}\exp.\left(\frac{i\theta}{2}\right)$$
$$= 2a\cos\left(\frac{\theta}{2}\right)\exp.\left\{i\left(\omega t - \alpha_1 + \frac{\theta}{2}\right)\right\} \qquad \ldots(5)$$

The magnitude of the resultant displacement y is given by

$$|y| = \sqrt{(y y^*)} = 2a\cos(\theta/2)$$

$$\therefore \quad \text{Intensity } I = |y|^2 = 4a^2\cos^2(\theta/2). \qquad \ldots(6)$$

Condition for maximum intensity

Intensity would be maximum (I_{max}) when

$$\cos\theta/2 = \pm 1$$

or

$$\theta/2 = n\pi$$

or

$$\frac{1}{2}\left[k\left(2x_1 + \frac{\lambda}{2}\right) - k\left(2x_2 + \frac{\lambda}{2}\right)\right] = n\pi \qquad \ldots(7)$$

In terms of path difference, intensity would be maximum when

$$\frac{2\pi}{\lambda}(2x_1 - 2x_2) = 2n\pi$$

or $2(x_2 - x_1) = n\lambda$ (when glass plate is heavily polished)

and $2(x_2 - x_1) = n\lambda \pm \frac{\lambda}{2}$ (when glass plate is lightly polished) …(8)

Condition for minimum intensity

Intensity would be minimum (I_{min}), when

$$\cos\frac{\theta}{2} = 0 \quad \text{or} \quad \frac{\theta}{2} = (2n+1)\frac{\pi}{2}$$

$$\frac{1}{2}\left[k\left(2x_1 + \frac{\lambda}{2}\right) - k\left(2x_2 + \frac{\lambda}{2}\right)\right] = (2n+1)\frac{\pi}{2}$$

In terms of path difference, intensity would be minimum when

$$\frac{2\pi}{\lambda}(2x_1 - 2x_2) = (2n+1)\pi$$

or $2(x_1 - x_2) = (2n+1)\frac{\lambda}{2}$ (when glass plate is heavily polished)

$2(x_1 - x_2) = (2n+1)\frac{\lambda}{2} \pm \frac{\lambda}{2}$ (when glass plate is lightly polished)

1.17.1 TYPES OF FRINGES:

1. Circular fringes:

In a Michelson's interferometer when the mirror M_1 is parallel to virtual image M_2^1 of the mirror M_2, circular fringes are produced with monochromatic light.

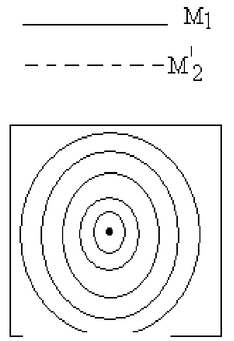


Fig. 1.25

2. Localized fringes:

When the mirror M_1 and virtual image M_2^1 are inclined at an angle, an air film is enclosed in wedge shape. The shape of the fringes for various cases are shown in figure. These fringes are known as localized fringes.

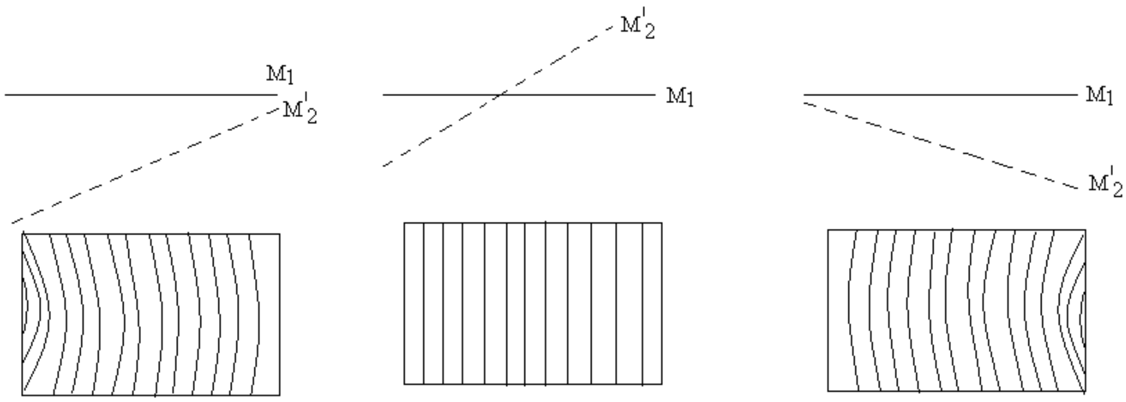


Fig. 1.26

3. Localized white fringes:

When monochromatic light source is replaced by a white source, then coloured and curved localized fringes are obtained (only when the air films has a small thickness). The fringes of zero thickness being again perfectly dark and straight. Other fringes are coloured due to overlapping of various colours. If the film is thick, uniform illumination is observed. White light fringes are useful for the determination of zero path difference, especially in the standardization of the meter.

1.17.2 USES OF MICHELSON'S INTERFEROMETER

Michelson's interferometer is used for a variety of purposes, for example:

(1) in the determination of wavelength of monochromatic source of light,

(2) to determine the difference between the two neighbor-hood wavelengths or resolution of the spectral lines

(3) in the determination of refractive index and thickness of various thin transparent materials,

(4) for the measurement of the standard meter in terms of the wavelength of light, etc.

UNIT-II
DIFFRACTION

2.1 INTRODUCTION

Bending of light rays from the edges of the objects and spreading of light into geometrical shadows of objects is wave nature of light. This phenomenon is known as "diffraction". Diffraction occurs near the edges of slits, apertures and other obstacles. Diffraction is divided into two classes:

Fresnel diffraction: The correct interpretation of diffraction was proposed by Fresnel. According to Fresnel, the diffraction phenomenon is due to the mutual interference of secondary wavelets origination from various points of the same wave front which are not blocked off by the obstacle. Here, the source of light & screen are at finite distance from the obstacle. The incident wave front is either spherical or cylindrical.

Fraunhofer diffraction: In this case of diffraction source of light & screen are at infinite distance from the obstacle. The incident wave front is plane and deals with parallel light rays. The conditions required for fraunhofer diffraction are achieved by using lenses. Fresnel diffraction is a general case and involves oblique angles of incidence while Fraunhofer diffraction simplifies diffraction to normal incidence. By using biconvex lenses, finite distances can be made infinite & hence in a Lab, one can achieve Fraunhofer condition from Fresnel conditions. Fraunhofer diffraction is a special case of Fresnel diffraction.

Interpretation of Diffraction by Fresnel:

Diffraction is due to mutual interference of secondary wavelets originating from various points of the wave front, which are not blocked off by an obstacle. Interference of secondary wavelets produces diffraction. *In Interference, interaction takes place between two separate wave fronts originating from two coherent sources while in diffraction interaction takes place between secondary wavelets originating from different points of the same wave front.* In interference pattern, regions of minimum intensity are usually perfectly dark while this is not the case in diffraction pattern. Maxima in diffraction do not have the same intensity like in interference.
Diffraction fringes are not equally spaced like in interference.

Applications

- ✓ Diffraction influences all segments of our daily lives.
- ✓ Diffraction being used in many fields of science and technology like physics, chemistry, medicine, biology, geology oil/gas industry, communication and detection systems to meet the needs of individual and society.
- ✓ X-ray diffraction is an effective approach to investigate the relationships that exist between the structure and properties of materials.

2.2 Differences between Fresnel's and Fraunhofer diffraction

Fresnel's diffraction	Fraunhofer diffraction
1. In Fresnel's diffraction the source and screen are placed at finite distance from the obstacle for producing diffraction pattern.	1. In Fraunhofer diffraction the source and screen are placed at infinite distance from the obstacle for producing diffraction pattern.
2. No lens is used to focus the rays.	2. Convex lens is used to focus the parallel rays.
3. The wave front undergoing diffraction is either spherical or cylindrical.	3. The wave front undergoing diffraction is a plane wave front.
4. The mathematical investigations are complicated and only approximate.	4. The mathematical investigations are easy and rigorous.
5. The centre of diffraction pattern may be bright or dark depending upon the number of Fresnel's zones.	5. The centre of diffraction pattern is always bright for all the paths parallel to the axis of the lens.

2.3 Differences between interference and diffraction

Interference	Diffraction
1. The interaction takes places between two different wave fronts originating from the coherent sources.	1. The interaction takes places between the secondary wavelets originating from different point of the same wave front.
2. The fringe width may or may not be equal.	2. The fringe width of various fringes are never equal.
3. All the bright fringes have the same intensity.	3. The bright fringes are of varying intensity
4. The regions of minimum intensity are perfectly dark, hence the fringes will appear with contrast.	4. The regions of minimum intensity are not perfectly dark, hence the fringes will not appear with contrast.

2.4 FRAUNHOFER DIFFRACTION AT SINGLE SLIT

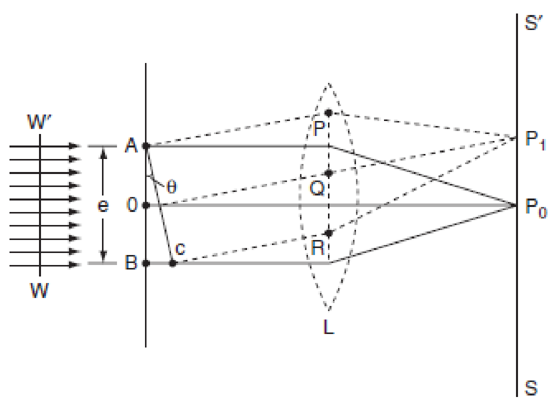


Fig. 2.1

Consider a plane wavefront WW′ of monochromatic light of wavelength λ incidenting normally on a section AB of a narrow slit. '*e*' be the width of the slit. Let the diffracted light be focused by means of a convex lens on a screen placed in the focal plane of the lens.

According to Huygens-Fresnel, every point of the wavefront is a source of secondary spherical wavelets, spreading in all directions radially. The secondary wavelets travelling normally to the slit, i.e., along the direction OP, are brought to focus at P_0 by the lens. Thus, P_0 is a bright central image. The secondary wavelets travelling at an angle θ with the normal are focused at a point P_1 on the screen. The point P_1 is of the minimum intensity or maximum intensity depending upon the path difference between the secondary waves from the corresponding points of the wavefront.

General Mathematical Theory

In order to find out intensity at P_1, draw a perpendicular AC on BR, the path difference between secondary wavelets from A and B in direction θ

$$BC = AB \sin\theta = e \sin\theta$$

and corresponding phase difference $\frac{2\pi}{\lambda} \text{e} \sin\theta$

Consider the width of the slit is divided into n equal parts and the amplitude of the wave from each part is *a*.

The phase difference between any two consecutive waves from these parts would be

$$\frac{1}{n}\left(\text{Total phase}\right) = \frac{1}{n}\left(\frac{2\pi}{\lambda}\text{e} \sin\theta\right) = d \ \left(say\right)$$

Using the method of vector addition of amplitudes the resultant amplitude R is given by

$$R = \frac{a \sin\frac{nd}{2}}{\sin\frac{d}{2}} = a\frac{\sin\left(\frac{\pi \text{e} \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi \text{e} \sin\theta}{n\lambda}\right)}$$

$$R = a\,\frac{\sin\alpha}{\sin\left(\alpha/n\right)} \qquad \text{where} \quad \alpha = \frac{\pi \text{e} \sin\theta}{\lambda}$$

$$= a\,\frac{\sin\alpha}{\left(\alpha/n\right)}$$

$$R = na\frac{\sin\alpha}{\alpha} = A\frac{\sin\alpha}{\alpha}$$

Thus, the resultant amplitude is given by $R = A\frac{\sin\alpha}{\alpha}$

when n→ ∞, *a* →0, but product n*a* = A (remains finite)

Now, the intensity is given by

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

Intensity distribution in single slit

1. Principal maximum

The expression for resultant amplitude R can be written in ascending powers of a as

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \dots\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \dots\right]$$

If the negative terms vanish, the value of R will be maximum, i.e.,

$$\alpha = \frac{\pi e \sin\theta}{\lambda} = 0 \quad \text{or} \quad \sin\theta = 0 \quad \text{or} \quad \theta = 0$$

Now, maximum value of R is A and intensity is proportional to A^2.

The maximum is known as principal maximum

2. Minimum

The intensity positions intensity will be minimum when $\sin\alpha = 0$. The values of a which satisfy this equation are

$$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \dots \text{etc} = \pm m\pi$$

$$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

$$e \sin\theta = \pm m\lambda$$

where m =1,2,3, ... etc.

In this way we obtain the points of minimum intensity on either side of the principal maximum. The value of m = 0 is not allowed, because when m = 0 then θ = 0 and this corresponds to principal maximun.

3. Secondary maxima

In addition to principal maximum at α = 0, there are weak secondary maxima between equally spaced minima. The positions can be obtained with the rule of finding maxima and minima of a given function in calculus. Differentiating the expression of/ with respect to a and equating to zero, we have

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$A^2\,\frac{2\sin\alpha}{\alpha}.\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

Here either $\sin\alpha = 0$ or $\alpha\cos\alpha - \sin\alpha = 0$

The equation α = 0 gives the values of α (except 0) for which the intensity is zero on the screen.

Hence, the positions of maxima are given by the roots of the equation

$$\alpha\cos\alpha - \sin\alpha = 0 \text{ or } \alpha = \tan\alpha. \qquad (4)$$

The values of α satisfying the above equation are obtained graphically by plotting the curves $y = \alpha$ and $y = \tan\alpha$ on the same graph.

The points of intersection of two curves give the values of α which satisfy eq. (2). The plots of $y = \alpha$ and $y = \tan\alpha$ are shown in Fig. 2.2.

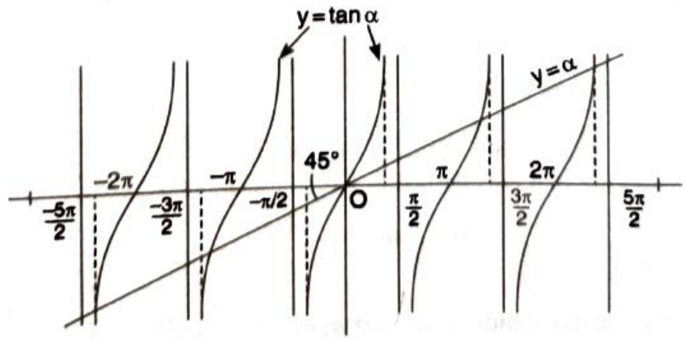


Fig. 2.2

The points of intersections are

$$\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2} \; etc.$$

$\alpha = 0$ gives principal maximum.

Substituting approximate values of α in eq. (2), we get the intensities in various maxima

$$I_0 = A^2 \left(Principal \; maxima\right)$$

$$I_1 = A^2\left[\frac{sin\;(3\pi/2)}{3\pi/2}\right] = \frac{A^2}{22} \; approx. \; \left(1^{st} \, subsidery \; maxima\right)$$

$$I_2 = A^2\left[\frac{sin\;(5\pi/2)}{5\pi/2}\right] = \frac{A^2}{62} \; approx. \quad \left(2^{nd} \, subsidery \; maxima\right)$$

and so on.

From the expressions of I_0, I_1, I_2, it is evident that most of the incident light is concentrated in the principal maximum.

4. Intensity distribution graph

A graph showing the variation of intensity with α is shown in fig. 2.3. The diffraction pattern consists of a central principal maximum occurring in the direction of incident rays. There are subsidiary maxima of decreasing intensity on either sides of it at positions $\alpha = \pm 3\pi/2, \pm 5\pi/2$ etc, and so on. Between subsidiary maxima, there are minima at positions $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \ldots etc = \pm m\pi$.

It should be noted that subsidiary maxima do not fall exactly mid-way between two minima, but they are displaced towards the centre of the pattern, of course, the displacement decreases as the order of maximum increases.

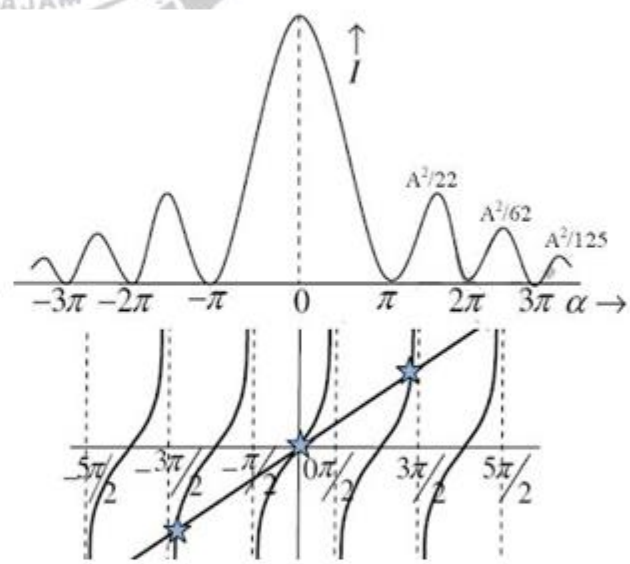


Fig. 2.3

Applications

- Fraunhofer diffraction is used to model the diffraction of waves when the diffraction pattern is viewed at a long distance from the diffracting object.
- The Fraunhofer Diffraction technique is used widely for particle sizing applications in the 1–200 μm size range.
- Fraunhofer diffraction method is used to study the plasma density fluctuations in high-temperature plasmas using infrared lasers, namely (i) development of the general theory of the Fraunhofer diffraction method, (ii) measurements of fluctuations propagating in an azimuthal direction, (iii) measurements of fluctuation intensities, and (iv) application in measurements on high-temperature plasmas.

2.5 RAYLEIGH'S CRITERION OF RESOLUTION

According to Rayleigh criterion, two sources are resolvable by an optical instrument when ***the central maximum in the diffraction pattern of one falls over the first minimum in the diffraction of the other and vice versa***. Similarly, in case of spectral lines of two different wavelengths, the lines will be resolved when the central maximum due to one wavelength falls over the first minimum due to other and vice versa.

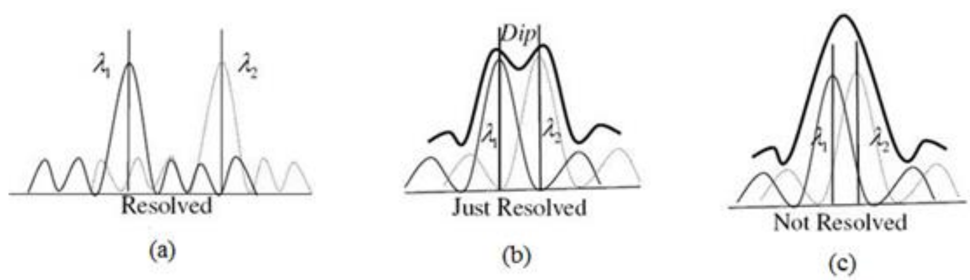


Fig. 2.4

In order to illustrate the criterion let us consider the resolution of two wavelengths λ and λ+dλ by a grating. Fig. 2.4 (a) shows the intensity curves of the diffraction patterns of two wavelengths. *The difference in wavelengths is such that their principal maxima are separately visible.* There is a distinct point of zero intensity in between the two. Hence, the <u>two wavelengths are resolved</u>. Now, consider the case when the difference in wavelengths is smaller and such that *the central maximum of wavelengths coincides with the first minimum of the other* as shown in fig. 2.4 (b). The resultant intensity curve is shown by thick curve. The curve shows a distinct dip in the middle of two central maxima, i.e., there is a noticeable decrease in intensity between the two central maxima of two different wavelengths. Thus, the two wavelengths can be distinguished from one another and according to Rayleigh they are said to <u>just resolved</u>.

Again consider the case when the difference in wavelengths is so small that the central maxima corresponding to two wavelengths come still closer as shown in fig. 2.4 (c). The resultant intensity curve in this case is quite smooth without any dip thus giving the impression as if there is only one wavelength source although somewhat bigger and stronger. Hence, the two wavelengths are not resolved.

Thus, the two spectral lines can be resolved only upto a certain limit expressed by Rayleigh criterion.

2.6 PLANE DIFFRACTION GRATING (NORMAL INCIDENCE)

Construction

An arrangement contains large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed very closely side by side at regular intervals. The diameters of the wires were of the order of 0.05 mm and their spacing, varied from 0.0533 mm to 0.687 mm.

Now, gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as slit. This is known as *plane transmission grating*.

On the other hand, if the lines are drawn on a silvered surface (plane or concave) then light is reflected from the positions of mirrors in between any two lines. It is called *reflection plane or concave grating*. When the spacing between the lines is of the order of the wavelength of light, then an appreciable deviation of the light is produced.

Theory

Fig. 2.5 represents the section of a plane transmission grating placed perpendicular to the plane of the paper. Let *e* be the width of each slit and *d* the width of each opaque part. Then (*e*+*d*) is known as grating element. XY is the screen placed perpendicular to the plane of a paper. Suppose a parallel beam of monochromatic light of wavelength λ be incident normally on the grating. By Huygen's principle, each of the slit sends secondary wavelets in all directions. The secondary wavelets travelling in the same direction of incident light will come to a focus at a point P_0 of the screen as the screen is placed at the focal plane of the convex lens. (The point P_0 will be a central maximum.)

Now, consider the secondary waves travelling in a direction inclined at an angle θ with the direction of the incident light. These waves reach point P_1 on passing through the convex lens in different phases. As a result dark and bright bands on both sides of central maximum are obtained.

The intensity at point P_1 may be considered by applying the theory of Fraunhofer diffraction at a single slit. The wavelets proceeding from all points in a slit along the direction are equivalent to a single wave of amplitude $\left(A\,\frac{\sin\alpha}{\alpha}\right)$ starting from the middle point of the slit, where $\alpha = \frac{\pi e \sin\theta}{\lambda}$.

If there are N slits, then we have N diffracted waves, one each from the middle points of the slits. The path difference between two consecutive slits is $\left(e+d\right)\, sin\ \theta$. Therefore, there is a corresponding phase difference $\left(2\pi/\lambda\right)\ \left(e+d\right)\ sin\ \theta$ between the two consecutive waves. The phase difference is constant and it is 2 β.

Hence, the problem become to find the resultant amplitude of N vibrations each of amplitude $\left(A\,\frac{\sin\alpha}{\alpha}\right)$ and having a common phase difference

$$\left(2\pi/\lambda\right)\ \left(e+d\right)\ sin\ \theta = 2\beta \qquad (1)$$

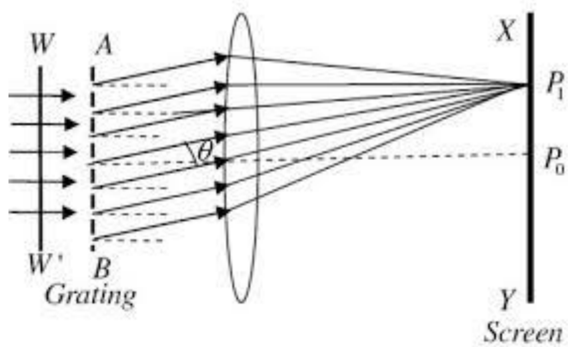


Fig. 2.5

By the method of vector addition of amplitudes,

$$R = a\frac{\sin {}^{nd}\!/_2}{\sin {}^{d}\!/_2}$$

In the present case, $a = A\dfrac{\sin\alpha}{\alpha}$, $n = N$ and $d = 2\beta$

$$R = \mathrm{A}\frac{\sin\alpha}{\alpha}\ \frac{\sin N\beta}{\sin\beta}$$

And $$I = R^2 = \mathrm{A}^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \frac{\sin^2 N\beta}{\sin^2\beta}$$

The factor $\left(\mathrm{A}\dfrac{\sin\alpha}{\alpha}\right)^2$ gives the distribution of intensity due to a single slit while the factor $\dfrac{\sin^2 N\beta}{\sin^2\beta}$ gives the distribution of intensity as a combined effect of all the slits.

<u>Intensity distribution in grating</u>

<u>Principal maxima</u>

The intensity would be maximum when $\sin\beta = 0$

or $\beta = \pm\ n\pi$ where $n = 0,1,2,3,...$

But at the same time $\sin N\beta = 0$, so that the factor $\dfrac{\sin N\beta}{\sin\beta}$ becomes indeterminate. It may be evaluated by applying the Hospital's rule. Thus,

$$\lim_{\beta\to\pm n\pi}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}\left(\sin\beta\right)}$$

$$= \lim_{\beta\to\pm n\pi}\frac{N\ \cos N\beta}{\cos\beta} = \pm N$$

Hence $$\lim_{\beta\to\pm n\pi}\left(\frac{\sin N\beta}{\sin\beta}\right)^2 = N^2$$

The resultant intensity is N^2, The maxima are most intense and are called as principal maxima.The maxima are obtained for

$$\beta = \pm\, n\pi$$

$$\pi/\lambda\ (e+d)\ sin\ \theta\ =\ n\ \pi$$

$$or\quad (e+d)\ sin\ \theta\ =\ n\lambda \qquad \text{where } n\ = 0,1,2,3,...$$

n = 0 corresponds to zero order maximum. For n =1,2,3,... etc., we obtain first, second, third, etc., principal maxima respectively. The $\pm$ sign shows that there are two principal maxima of the same order lying on either side of zero order maximum.

Minima

If sin Nβ = 0 but sin β ≠ 0

For minima sin Nβ = 0,

$$N\beta = \pm\, m\pi$$

$$N\ \pi/\lambda(e+d)\ sin\theta\ = \pm\, m\pi$$

A series of minima occur, when N *(e+d)* sin θ = ± mλ

where m has all integral values except 0, N, 2N, ...nN, because for these values sin β becomes zero and we get principal maxima. Thus, m = 1,2,3,... (N -1). Hence, there are adjacent principal maxima.

Secondary maxima

As there are (N -1) minima between two adjacent principal maxima there must be (N -2) other maxima between two principal maxima. To find out the position of these secondary maxima, we differentiate equation (3) with respect to β and then equate it to zero. Thus,

$$\frac{dI}{d\beta} = \frac{d}{d\beta}\left[\left(A\frac{\sin\alpha}{\alpha}\right)^2 \frac{\sin^2 N\beta}{\sin^2\beta}\right] = 0$$

$$= \left(\frac{A\sin\alpha}{\alpha}\right)^2 2\left(\frac{\sin N\beta}{\sin\beta}\right)\left[\left(N\cos N\beta\sin\beta - \sin N\beta\cos\beta\right)/\sin^2\beta\right] = 0$$

$$N\cos N\beta\sin\beta - \sin N\beta\cos\beta = 0$$

$$N\tan\beta = \tan N\beta$$

The roots of this equation other than those for which $\beta = \pm n\pi$ (which correspond to principal maxima) give the positions of secondary maxima. To find out the value of $\frac{\sin N\beta}{\sin\beta}$ from equation $N\tan\beta = \tan N\beta$, we make use of the triangle shown in fig. 2.6.

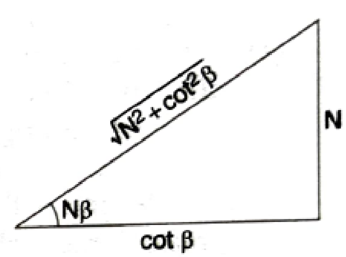


Fig. 2.6

From fig. 2.6,

$$\sin N\beta = \frac{N}{\sqrt{N^2 - \cot^2\beta}}$$

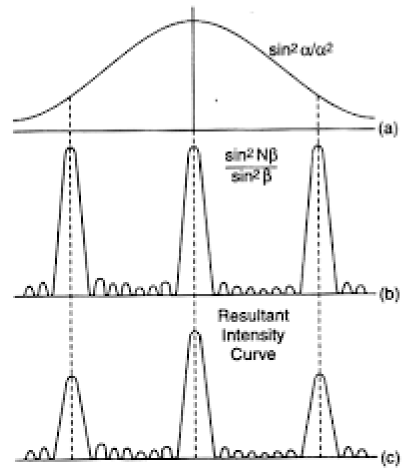
$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{(N^2 - \cot^2\beta)\times \sin^2\beta} = \frac{N^2}{(N^2 \sin^2\beta + \cot^2\beta)}$$

$$= \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

$$\frac{\text{Intensity of secondry maxima}}{\text{Intensity of principal maxima}} = \frac{1}{1+(N^2-1)\sin^2\beta} \qquad (4)$$

As N increases, the intensity of secondary maxima relative to principal maxima decreases and becomes negligible when N becomes large.

The resultant intensity distribution is complex and shown in fig 2.7.



2.7 DETERMINATION OF WAVELENGTH USING DIFFRACTION GRATING: NORMAL INCIDENCE

Theory

The diffraction grating is often used in the laboratories for measuring wavelength of light. In a diffraction grating, the principal maxima are obtained in the directions given by

$$(e+d)\sin\theta = n\lambda \qquad (1)$$

where (e+d) is the grating element, n is the order of maximum and θ is the angle of diffraction corresponding to a particular wavelength. The number of lines N ruled on the grating (per inch) are written over it by the manufacturers. Hence,

$$N(e+d) = 1'' = 2.54 \text{ cm or } e+d = (2.54/N) \text{ cm}$$

Thus, the determination of wavelength involves the measurement of angle of diffraction θ, for a given wavelength in a particular order n. In the laboratory, the grating spectrum of a given source of light (monochromatic or polychromatic) is obtained by using a spectrometer.

Adjustments

Before performing the experiment, the following adjustments are made:

(1) The spectrometer is adjusted for parallel rays by Schuster's method.

(2) The grating is adjusted for normal incidence.

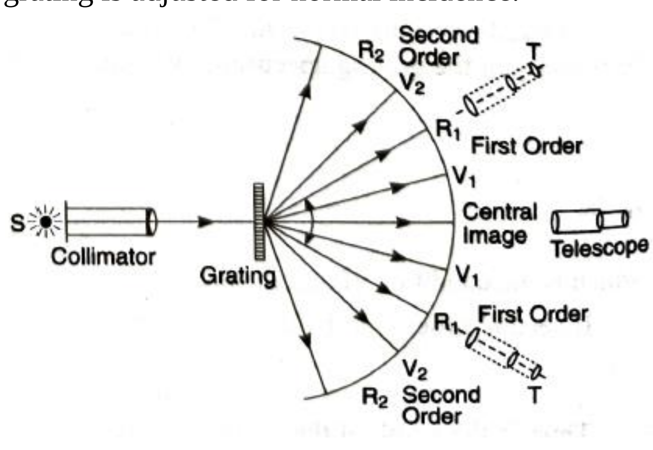


Fig. 2.8

a) The slit of the collimator is illuminated by the given source of light. Now, the position of the telescope is adjusted in such a way that the image of the slit is focused on the vertical cross-wire of the telescope. In this position the collimator and the telescope are in the same line.
b) The position of the telescope is noted on the circular scale. It is now turned to 90° and clamped. The given transmission grating is mounted at the centre of the prism table such that the grating surface is perpendicular to the prism table.
c) The prism table is now rotated so that the image of the slit reflected from the grating surface lies at the intersection of the cross-wires. In this position the grating is at 45° to the incident light.
d) The prism table is suitably rotated through 45° in such a way that the grating is exactly normal to the incident light. The prism table is clamped.

Measurement of θ.

(i) When the source of light emits radiations of different wavelengths, then the beam gets dispersed by the grating and in each order a spectrum of the different wavelengths is observed.
(ii) The telescope is now turned to get the first order spectrum. The cross-wire is adjusted on the line for which wavelength is to be determined (say Red). The position of telescope (T) is shown by dotted lines in fig. 2.8. The readings of the two Vernier's are recorded.
(iii) The telescope is then turned to go to the first order on the other side and the cross-wire is adjusted on the same coloured line (Red). The position of telescope (T) is shown by dotted lines. The readings of two Vernier are again recorded.
(iv) The difference between readings of the same Vernier gives twice the angle of diffraction for that line in first order. By substituting the value of θ in equation (1), the wavelength of light can be calculated.
(v) The same observation may be repeated in second order and so on. By following the same procedure, the wavelengths of different lines can be measured with the help of grating.

2.8 DETERMINATION OF WAVELENGTH USING DIFFRACTION GRATING OBLIQUE INCIDENCE

Fig-2.9 (a) represents the section of a plane transmission grating placed perpendicular to the plane of the paper. Let e and d be the width of each slit and opaque part. Then (e+d) is known as grating element. Let rays of light be incident at an angle i on N-slit system.

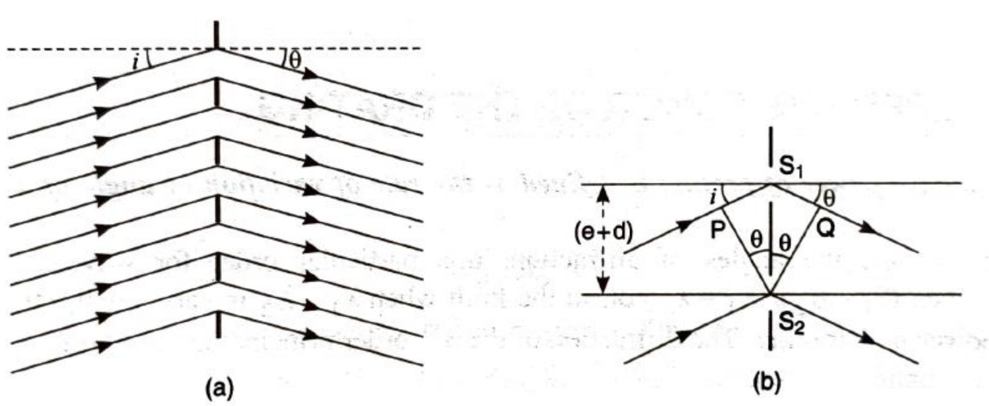


Fig. 2.9

Now, we shall calculate the path difference between the secondary wavelets diffracted at an angle through any two consecutive slits as shown in fig. 2.9 (b). In fig. 2.9 (b), S_1 and S_2 are the middle points of two adjacent slits. From S_2, we draw two perpendiculars S_2P and S_2Q. The path difference is given by

$$PS_1 + S_1Q = (e+d)\sin i + (e+d)\sin\theta$$

$$PS_1 + S_1Q = (e+d)[\sin i + \sin\theta] \qquad \ldots(1)$$

The condition for n^{th} order principal maximum is

$$(e+d)[\sin i + \sin\theta_n] = n\lambda$$

$$(e+d)\left[2\sin\left(\frac{\theta_n + i}{2}\right)\cos\left(\frac{\theta_n - i}{2}\right)\right] = n\lambda$$

$$\sin\left(\frac{\theta_n + i}{2}\right) = \frac{n\lambda}{2(e+d)\cos\left(\frac{\theta_n - i}{2}\right)}. \qquad \ldots(2)$$

The deviation ($\theta_n + i$) will be minimum when $\sin\left(\frac{\theta_n + i}{2}\right)$ minimum. It is clear from eq. (2) that $\sin\left(\frac{\theta_n + i}{2}\right)$ will be minimum when $\cos\left(\frac{\theta_n - i}{2}\right)$ is maximum. Therefore

$$\theta_n - i = 0 \quad \text{or} \quad \theta_n = i \qquad \ldots(3)$$

So, the deviation produced in the diffracted beam from the incident direction will be minimum when the angle of diffraction θ_n, is equal to the angle of incidence *i*.

Therefore, the angle of minimum deviation D_m is given by

$$D_m = \theta_n + i = \theta_n + \theta_n = 2\theta_n$$

$$i = \theta_n = \frac{D_m}{2} \qquad \ldots(4)$$

The condition for principal maxima now becomes

$$(e+d)\left[\sin\left(\frac{D_m}{2}\right) + \sin\left(\frac{D_m}{2}\right)\right] = n\lambda$$

$$2(e+d)\sin\left(\frac{D_m}{2}\right) = n\lambda. \qquad \ldots(5)$$

For minimum deviation condition, using eq. (5), we can determine the wavelength of light using a grating in minimum deviation position.

2.9 RESOLVING POWER OF A GRATING

One of the important properties of a diffraction grating is its ability to separate spectral lines which have nearly the same wavelength. The resolving power of a diffraction grating is defined as the capacity to form separate diffraction maxima of two wavelengths which are very close to each other.

This is measured by $\lambda/d\lambda$, where $d\lambda$ is the smallest difference in two wavelengths which are just resolvable by grating and λ is the wavelength of either of them or mean wavelength.

Expression for resolving power

Let AB represent the surface of a plane transmission grating having grating element (*e*+*d*) and N total number of slits. Let a beam of light having two wavelengths λ and $\lambda+d\lambda$ be normally incident on the grating. In fig, XY is the field of view of the telescope, P_1 is n^{th} primary maximum of a spectral line of wavelength λ at an angle of diffraction θ_n and P_2 is the n^{th} primary maximum of wavelength $(\lambda+d\lambda)$ at diffracting angle $(\theta+d\theta)$.

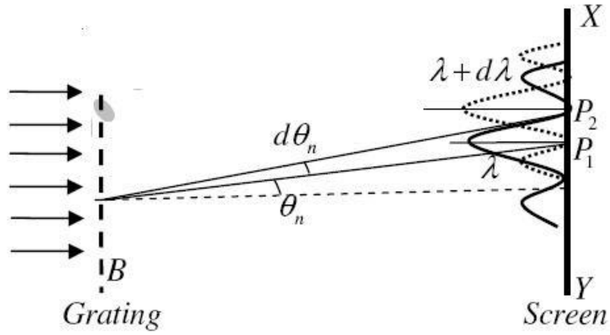


Fig. 2.10

According to Rayleigh criterion, the two wavelengths can be resolved if the position of P_2 corresponds to the first minimum of P_1, i.e., the two lines will be resolved if the principal maximum of $(\lambda+d\lambda)$ [in n^{th} order] in a direction $(\theta_n+d\theta_n)$ falls over the first minimum of λ in the same direction $(\theta_n+d\theta_n)$. Now, we shall consider the first minimum of λ in the direction $(\theta_n+d\theta_n)$ in the following way:

The principal maximum of λ in the direction θ_n is given by

$$(e+d)\sin\theta_n = n\lambda \quad \ldots(1)$$

The equation of minima is

$$N(e+d)\sin\theta = m\lambda,$$

where m has all integral values except 0, N, 2N,..., nN, because for these values we obtain different maxima.

Thus, first minimum adjacent to n^{th} principal maximum in the direction $(\theta_n+d\theta_n)$ can be obtained by substituting the value of m as (nN+1).

Therefore, first minimum in the direction $\theta_n+d\theta_n$ is given by

$$N(e+d)\sin(\theta_n + d\theta_n) = (nN+1)\lambda \quad \ldots(2)$$

The principal maximum of $(\lambda+d\lambda)$ in direction $(\theta_n+d\theta_n)$ is given by

$$(e+d)\sin(\theta_n + d\theta_n) = n(\lambda + d\lambda) \quad \ldots(3)$$

Multiplying eq. (3) by N, we have

$$N(e+d)\sin(\theta_n + d\theta_n) = nN(\lambda + d\lambda) \quad \ldots(4)$$

From eqs. (2) and (4), we get

$$(nN+1)\lambda = nN(\lambda + d\lambda)$$
$$nN\lambda + \lambda = nN\lambda + nN\,d\lambda$$
$$\lambda = nN\,d\lambda$$
$$\frac{\lambda}{d\lambda} = nN.$$

This is the required expression.

Thus, the resolving power is directly proportional to (i) the order of the spectrum and (ii) the total number of lines on the grating surface.

From eq. (1), $$n = \frac{(e+d)\sin\theta_n}{\lambda}$$

$$\therefore \quad \frac{\lambda}{d\lambda} = \frac{N(e+d)\sin\theta_n}{\lambda}. \quad \ldots(6)$$

2.10 DIFFERENCE BETWEEN DISPERSIVE POWER AND RESOLVING POWER OF A GRATING

Here, we would like to mention the difference between dispersive power and resolving power of a grating. The differences are as follows:

1. The dispersive power of grating gives an idea of angular separation between two lines produced by grating while resolving power tells the limit of just resolution of two closer objects.
2. Dispersive power is measured by dθ/dλ (dθ being the angular separation between the lines). Its value is given by $\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\,sin\theta}$.

   The resolving power is measured by λ/dλ and is given by $\frac{\lambda}{d\lambda} = nN$.
3. When N (number of lines on grating surface) is increased, the dispersive power remains unchanged while resolving power is increased.
4. If (e+d) [the grating element] is decreased, the dispersive power is increased while resolving power remains unchanged.

2.11 HUYGEN's-FRESNEL THEORY OF LIGHT PROPAGATION

According to wave theory of light, each point in a light source sends out waves in all directions. Now the ether particles (hypothetical medium) are set up in vibration. The locus of all the ether particles vibrating in the same phase is called a wavefront.

Further, every point on the wavefront sends out secondary wavelets. Fresnel assumed that these wavelets are in a position to interfere. The resultant intensity at any point is the result of interference of these wavelets. The resultant intensity at any point due to wavefront can be calculated by dividing it into a number of zones which are called as 'Fresnel's half period zones'.

2.12 FRESNEL'S HALF PERIOD ZONES

Let ABCD [Fig. 2.11] be a monochromatic plane wavefront of wavelength λ proceeding in the direction of arrow and the resultant intensity is calculated at P. The complexity in calculating the resultant intensity was simplified by Fresnel by dividing the wavefront into a number of zones, known as Fresnel's half-period zones.

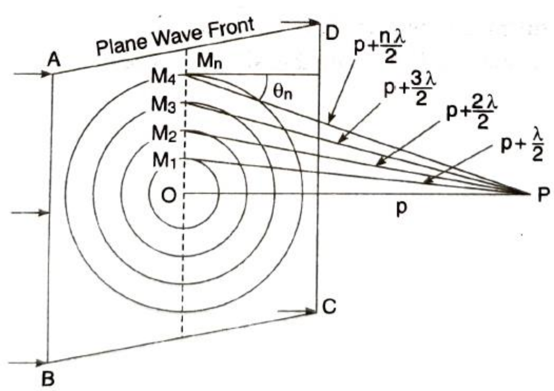


Fig. 2.11

Let PO be the normal to the wavefront (PO = p say). With P as the centre and radii equal to $p + \lambda/2, p + 2\lambda/2, \ldots p + n\lambda/2$ spheres are drawn. The plane ABCD cuts these spheres in concentric circles with center O and radii $OM_1, OM_2, \ldots, OM_n$. The area of the first innermost circle is the first half-period zone. Similarly, the areas enclosed between first and second circles, second and third circles, (n-1) and nth circles are known as 2nd, 3rd,..., half-period zones respectively. It is assumed that a resultant wave starts from each zone.

1. Relative Phases of the zones

As $M_1P = OP + \lambda/2$, hence the ray reaching at P from O and the circumference of the circle of radius OM_1 are in opposite phase, i.e., there is a path difference of $\lambda/2$ or a phase difference of π. The phase difference due to other points lying in the circle of radius OM_1 will vary from 0 to π. Thus, the mean phase of the secondary wavelets originated from first zone may be taken as $(0+\pi)/2 = \pi/2$. The phase due to M_1 and M_2 will differ by π, thus, the mean phase of the secondary wavelets originated from the second zone is $(\pi + 2\pi)/2 = 3\pi/2$. Similarly, it can be shown that mean phase due to third, fourth, etc., zones are $5\pi/2$, $7\pi/2$, etc. respectively. The successive zones differ in phase by π or by a half period (T/2), that is why they are called as half-period zones.

2. Amplitude due to a zone

The amplitude of disturbance at P due to the wave from a zone varies:

(i) directly as the area of the zone,

(ii) Inversely as the average distance of the zone from P,

(ii) Directly as $f(\theta_n)$, where $f(\theta_n)$ is a function of the θ_n. θ_n is a measure of the obliquity of the nth zone.

We will consider these points one by one.

(i) The area of the first half period zone

The different zones, for the purpose of calculating the area are shown in fig

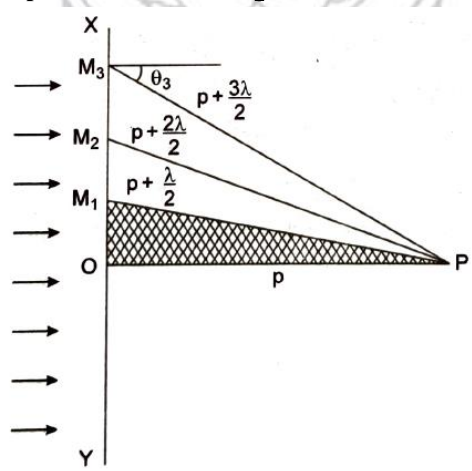


Fig. 2.12

The area of the half period zone can be calculated from

$$\pi(OM_1)^2 = \pi\left[(M_1P)^2 - (OP)^2\right] = \pi\left[\left(p+\frac{\lambda}{2}\right)^2 - p^2\right]$$

$$= \pi\left[p\lambda + \frac{\lambda^2}{4}\right] = \pi\, p\lambda \qquad approx.$$

The radius OM_1, of the first half period zone is $\sqrt{p\lambda}$

The radius OM_2 of the second half period zone is given by

$$OM_2 = [(M_2P)^2 - (OP)^2]^{1/2} = [(p+\lambda)^2 - p^2]^{1/2} = \sqrt{2p\lambda} \qquad \textit{approx.}$$

The area of the second half period zone

$$= \pi[(OM_2)^2 - (OM_1)^2] = \pi[2p\lambda - p\lambda] = \pi p\lambda \qquad \textit{approx.}$$

Thus, the area of second half period is equal to $\pi p\lambda$, i.e., approximately the same.

In general, the radius of n^{th} zone = $\sqrt{n\pi\lambda}$

(ii) Average distance of a zone

Let us consider the average distance of the n^{th} zone from P

$$= \frac{\left(p + n\frac{\lambda}{2}\right) + \left\{p + (n-1)\frac{\lambda}{2}\right\}}{2}$$

$$= p + (2n-1)\frac{\lambda}{4}$$

(iii) Obliquity factor θ_n

This is the angle between the normal to the zone and the line joining the zone to P. The obliquity factor is denoted by $f(\theta_n)$

$$\text{Amplitude of } n^{th} \text{ zone is} \propto \frac{\pi\left[p + (2n-1)\frac{\lambda}{4}\right]\lambda}{p + (2n-1)\frac{\lambda}{4}} f(\theta_n)$$

$$\propto \pi\lambda\, f(\theta_n)$$

As the order of the zone increases $f(\theta_n)$ decreases. Thus, the amplitude of wave from a zone at P decreases as n increases.

3. Resultant amplitude

Let R_1, R_2, R_3, … be the amplitudes of the waves at P due to the secondary waves from the first, second, third, ...etc. half period zones respectively.

As discussed above, the amplitude decreases as n increases, because obliquity decreases, hence R_1, R_2, R_3, ... continuously decreasing order. R_1 is slightly greater than R_2, R_2 is slightly greater than R_3, and so on.

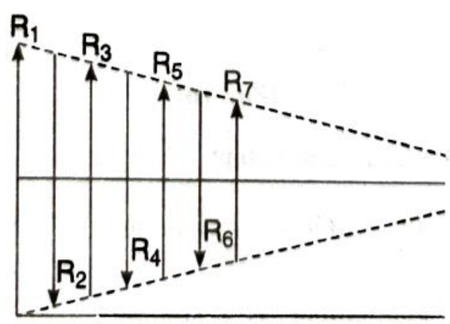


Fig. 2.13

As the amplitudes are gradually decreasing in magnitude, the amplitude of any vibration at P due to any Zone can be approximately taken as the mean of the amplitudes due to the zones preceding and succeeding it.

Hence, $R_2 = (R_1 + R_3)/2$. The successive amplitudes are shown in reverse directions as there is a phase difference of π between any two consecutive zones.

The resultant amplitude at P at any instant is given by

$$R = R_1 - R_2 + R_3 - R_4 + \ldots + R_n \quad \text{if n is odd}$$
$$= R_1 - R_2 + R_3 - R_4 + \ldots - R_n \quad \text{if n is even}$$
$$R = \frac{R_1}{2} + \left[\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right] + \left[\frac{R_3}{2} - R_4 + \frac{R_5}{2}\right] + \ldots$$
$$= \frac{R_1}{2} + \frac{R_n}{2} \qquad \left[\because R_2 = \frac{R_1 + R_2}{2}\right]$$

Considering n as even

$$R = \frac{R_1}{2} + \frac{R_{n-1}}{2} - R_n$$

As $n \to \infty$, R_n and R_{n-1} tend to zero as amplitudes are gradually diminishing. Therefore, the resultant amplitude at P due to the whole wavefront $R_1/2$.

The resultant intensity $I \propto \left(\frac{R_1}{2}\right)^2 \propto \frac{(R_1)^4}{4}$

2.13 RECTILINEAR PROPAGATION OF LIGHT

Let a plane wave front of monochromatic light be incident normally on a circular aperture. Let a screen be placed parallel to the aperture at some distance.

Consider a point P_1 (well inside the full line circle) on the screen. O_1 is the corresponding pole on the aperture. It is clear from the figure that a sufficient number of zones are exposed and hence the resultant amplitude at P_1 is $R_1/2$ in accordance with the above given Fresnel's theory.

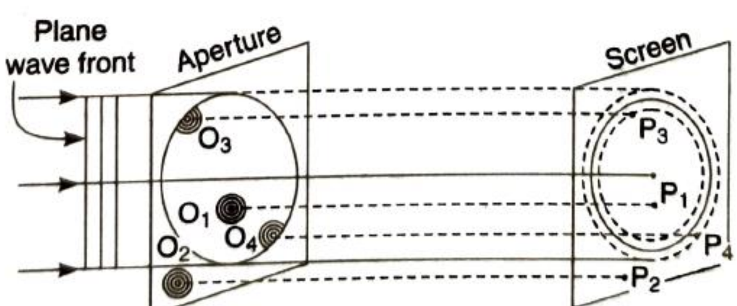


Fig. 2.14

.

Thus, a uniform illumination around P_1 is obtained.

Similarly, for a point P_2 which is within the geometrical shadow, a large number of effective zones are blocked and hence the resultant amplitude is zero. Thus, complete darkness is observed around P_2.

For points such as P_3 and P_4, within dotted circles on the screen, the corresponding poles are O_3 and O_4, and the areas of effective zones changes for the theory to apply. Thus, Fresnel's theory fails in explaining the rectilinear propagation near the edge. In this way the phenomenon of rectilinear propagation is explained on the basis of wave theory but only approximately.

2.14 ZONE PLATE

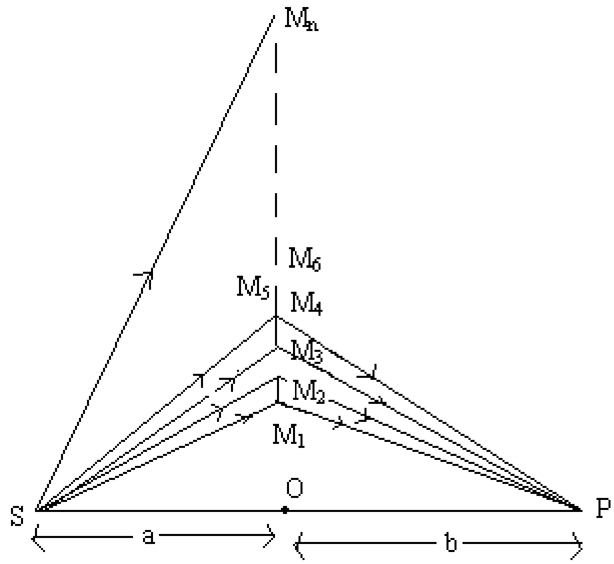


Fig. 2.15

The correctness of Fresnel's method of dividing a plane wavefront into half period zones, which forms the basis of the proof of the rectilinear propagation of light, can be verified by means of an optical device known as zone plate. It is specially constructed diffraction screen such that light from alternating zones are cut off.

Construction:

A large number of concentric circles, with radii is directly proportional to square root of natural numbers, are drawn on a sheet of white paper. The odd number of zones are painted black. Photograph of this pattern is taken on a glass plate. In the negative of the photograph the odd zone which are painted black are transparent and even zones appear opaque. This negative is called positive zone plate.

If the even zones are transparent and odd zones are opaque then the negative is called negative zone plate.

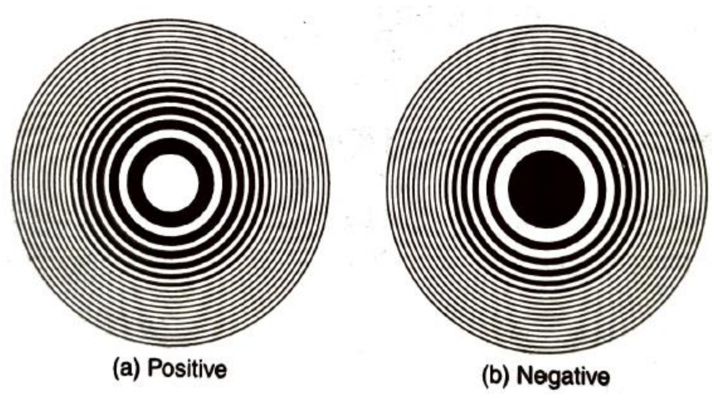


Fig. 2.16

Theory:

S is a point source of monochromatic light of wavelength λ. In the figure 'O' is called centre of the zone plate. OM_n represents a positive zone plate. From first zone to the next zone there is an increasing path difference of $\frac{\lambda}{2}$.

$$SM_1 + M_1P = SO + OP + \frac{\lambda}{2} = a + b + \frac{\lambda}{2}$$

$$SM_2 + M_2P = SO + OP + \frac{2\lambda}{2} = a + b + \frac{2\lambda}{2}$$

$$SM_3 + M_3P = SO + OP + \frac{3\lambda}{2} = a + b + \frac{3\lambda}{2}$$

....

....

$$SM_n + M_nP = SO + OP + \frac{n\lambda}{2} = a + b + \frac{n\lambda}{2} \qquad ------(1)$$

From Δle SOM_n, $SM_n{}^2 = SO^2 + OM_n{}^2$

$$= a^2 + r_n^2$$

$$SM_n = \sqrt{a^2 + r_n^2}$$

$$= \left(a^2 + r_n^2\right)^{1/2}$$

$$= a\left(1 + \frac{r_n^2}{a^2}\right)^{1/2}$$

$$= a\left(1 + \frac{r_n^2}{2a^2}\right)$$

$$\therefore SM_n = a + \frac{r_n^2}{2a}$$

similarly $M_nP = b + \frac{r_n^2}{2b}$

Substitute SM_n and M_nP in equation (1)

$$a + \frac{r_n^2}{2a} + b + \frac{r_n^2}{2b} = a + b + \frac{n\lambda}{2}$$

$$r_n^2\left(\frac{1}{a} + \frac{1}{b}\right) = n\lambda$$

In the above equation if a, b and λ are constant

By applying sign convention

$$r_n^2\left(-\frac{1}{a} + \frac{1}{b}\right) = n\lambda$$

$$\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{n\lambda}{r_n^2} = \frac{1}{f_n}$$

$$f_n = \frac{r_n^2}{n\lambda}$$

It is similar to a lens formula

$$\left(\frac{1}{v} - \frac{1}{u}\right) = \frac{1}{f}$$

v = b as image distance, $u = a$ as object distance, $f = f_n$, as the focal length of the lens.

Now, comparing the two, we have

$$f_n = \frac{r_n^2}{n\lambda}$$

This gives the focal length of zone plate. Thus, the zone plate behaves like a convergent lens.

2.15 COMPARISON BETWEEN A ZONE PLATE AND A CONVEX LENS

Similarities

(i)Both form real image of an object on the side other than the object. The distances of the object and image are connected together by similar formulae in both the cases.

(ii) Focal length for both depends upon λ and hence both show chromatic aberration

For zone plate, $$f_n = \frac{r_n^2}{n\lambda}$$

For convex lens, $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Dissimilarities

(i) In case of convex lens, the rays are brought to focus by refraction while in case of a zone plate, the image is formed by diffraction.

(ii) In case of convex lens, all the rays reaching the image point have the same optical path while in a zone plate, waves reaching the image point through two alternate zones differ in path by λ.

(iii) The image due to a convex lens is more intense than due to a zone plate.

(iv) For a convex lens $f_V < f_R$, while for zone plate $f_R < f_V$

(v) A convex lens has only one focus on one side of it while a zone plate has multiple foci at distances $(r_n^2)/n\lambda, (r_n^2)/3n\lambda, (r_n^2)/5n\lambda, ...$ The intensity diminishes as the focal length decreases.

UNIT- III
POLARISATION

3.1 INTRODUCTION:

Polarization is a property of waves that describes the orientation of their plane of vibrations. Light waves are transverse electromagnetic waves. The electric and magnetic field vibrations are always perpendicular to the direction of propagation. However, in general the electric field vibrations are spread in all possible directions but still perpendicular to the direction of propagation. Getting only one kind of vibration of light from ordinary light is called polarization.

Representation of various types of light:

1. **Unpolarised light:**
   The ordinary light also called as unpolarised light, consists of a very large number of vibrations in all planes with equal probability at right angles to the direction of propagation. Hence, the unpolarised light is represented by a star as shown in fig. 3.1.

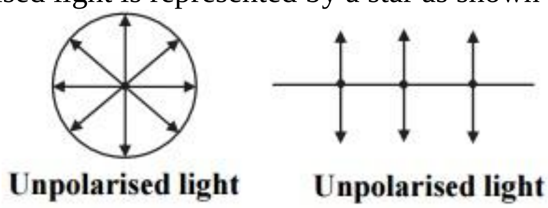


Fig. 3.1

2. **Plane polarised light:**
   We know that in plane polarised light the vibrations are along a straight line. If the direction of vibrations is parallel to the plane of paper, it is represented by a straight line arrow as shown in fig. 3.2 (A). If the direction of vibration is perpendicular to the plane of paper, it is represented by a dot as shown in fig. 3.2 (B).

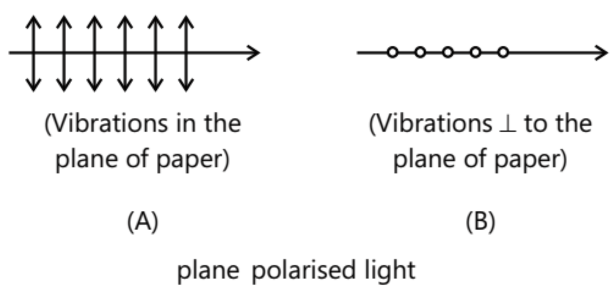


Fig. 3.2

3.2 PLANE OF POLARISATION:

When ordinary light is passed through a tourmaline crystal, the light is polarized and the vibrations are confined only in one directions which is perpendicular to the direction of propagation of light. Now, we consider the case of two planes: Firstly, the plane in which the vibrations of polarized light are confined. This plane is known as plane of vibrations as shown in figure…This plane contains the direction of vibrations as well as the direction of propagation. Secondly, the plane which has no vibrations. This plane is known as plane of polarization as shown in Fig. 3.3.

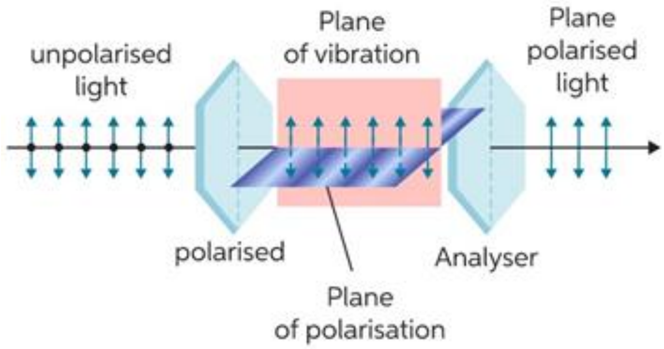


Fig. 3.3

Thus, a plane passing through the direction of propagation and perpendicular to the plane of vibration is known as plane of polarization.

3.3 TYPES OF POLARIZATIONS:

The polarization of light can be classified into three types. They are

1. Plane polarization
2. Circular polarization
3. Elliptical polarization.

3.3.1. Plane polarization:

When the light wave propagates through any medium, if the electric component of light passing through the medium vibrates only along a single direction perpendicular to the direction of propagation, the wave is said to be plane polarized or linearly polarized.

The resultant electric vector E can be resolved into two rectangular components E_x and E_y. Thus the transverse electric vector may be regarded as superposition of two mutually perpendicular electric fields.

Hence $E = E_x + E_y$. E is linearly polarized if two electric vectors are mutually perpendicular with zero phase difference superimpose, the magnitude of the resultant vector E remain linear between X and Y directions during the course of propagation.

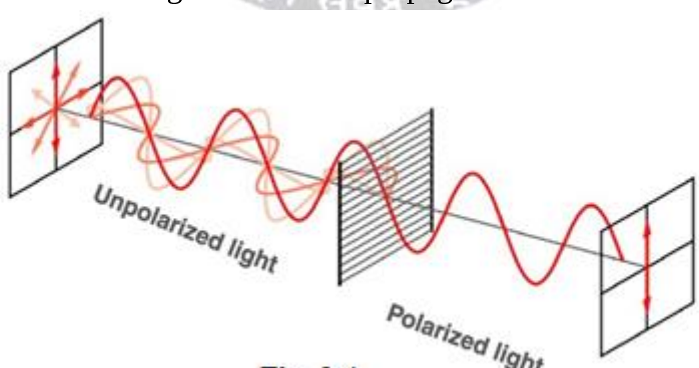


Fig. 3.4

3.3.2. Circular polarization:

When two coherent light waves of equal magnitudes and the electric vectors E mutually perpendicular to each other superimpose, the magnitude of the resultant vector E remain constant, but it rotating about the axis of direction of propagation such that it goes on sweeping a circular helix in space during the propagation. Such light wave is called circularly polarized light. If we imagine that we are looking into the light vector E, we observe that the tip of light vector E traces a circle on the plane perpendicular to the ray direction. If rotation of the vector tip is clockwise, the light is said to be right circular polarized or the tip is rotating anticlockwise, the light is said to be left circularly polarized.

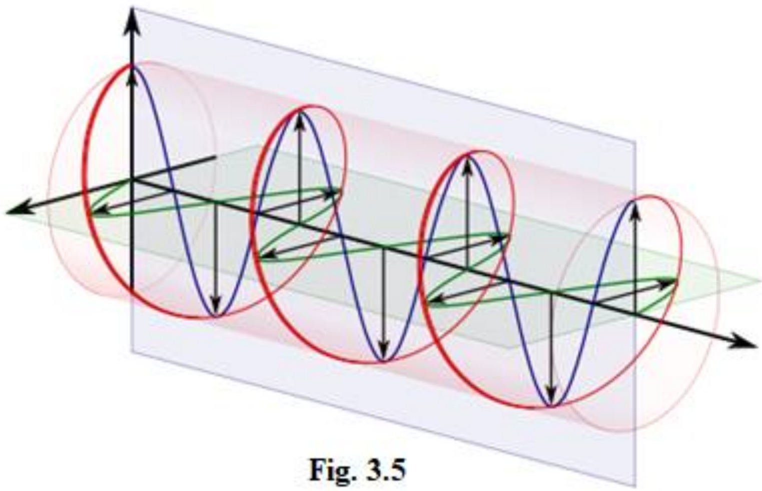


Fig. 3.5

3.3.3. Elliptical polarization:

When two coherent light wave of unequal magnitudes and different in phase superimposed, the magnitude of the electric vectors E changes with time and rotating about the axis of direction of propagation. The tip of the electric vector E sweeps a flattened helix in space and traces an ellipse in a plane perpendicular to the direction of propagation, such light is called elliptically polarized light.

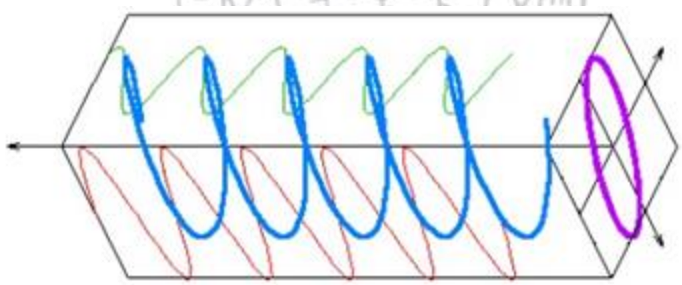


Fig. 3.6.

3.4 METHODS OF POLARISATION:

Following are the methods used for producing plane polarised light:

1. Polarisation by reflection.
2. Polarisation by refraction.
3. Polarisation by double refraction.
4. Polarisation by selective absorption.
5. Polarisation by scattering.

3.4.1 Polarisation by Reflection (BREWSTER'S LAW):

The simplest way of producing a plane polarised light is by reflection. In 1808, Malus discovered that when ordinary light is reflected from the surface of a transparent medium like glass or water it becomes partly polarised. The degree of polarisation changes with the angle of incidence. At a particular angle of incidence the reflected light has the greatest percentage of polarised light. The angle depends upon the nature of the reflecting surface. The angle of incidence is known as angle of polarisation.

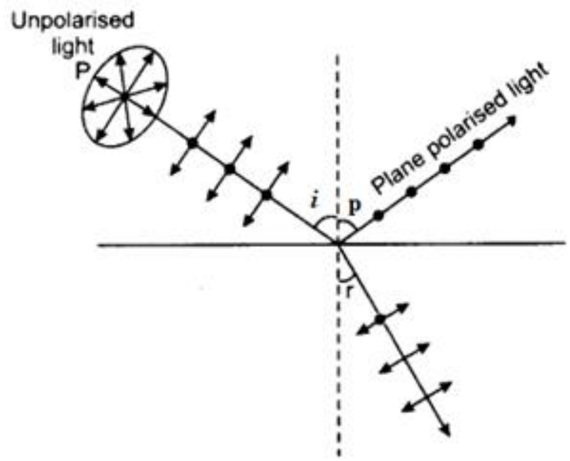


Fig. 3.7

In 1811, Brewster performed a number of experiments to study the polarisation of light by reflection at different surfaces. He observed that for a particular angle of incidence known as angle of polarisation, the reflected light is completely polarised in the plane of incidence. i.e., having plane of vibrations perpendicular to the plane of incidence.

Brewster proved that the tangent of the angle of polarisation is numerically equal to the refractive index of the medium, i.e.,

$$\mu = \tan p$$

This is known as Brewster's law. He also proved that the reflected and refracted rays are perpendicular to each other.

Angle between reflected and refracted rays:

Suppose a beam of unpolarised light is incident on glass surface at polarising angle p as shown in fig. 3.7. The polarising angle for air glass is 57°. A part of incident light is reflected while a part is refracted. Let r be the angle of refraction. From Brewster's law,

$$\mu=\tan p \qquad \ldots(1)$$

from Snell's law,

$$\mu =\sin p/\sin i \qquad \ldots(2)$$

comparing eqn. 1 and eqn. 2,

$$\tan p = \sin p/\sin i$$

$$\sin p/\cos p = \sin p/\sin r$$

$$\sin r = \cos p= \sin (90°-p)$$

$$r = 90°-p$$

$$r + p = 90°$$

Therefore the reflected and refracted rays are at right angles to each other. Here, it should be noted that the refracted index of a substance varies with wavelengths. Therefore, for complete polarisation the light should be monochromatic.

3.4.2 Polarisation by Refraction - Pile of Plates:

When ordinary light (unpolarised light) is incident on the upper surface of the glass slab at polarising angle, a small fraction is reflected and rest is refracted. The reflected light is completely plane polarised with vibrations perpendicular to the plane of incidence as shown in fig…The reflected light is partially polarised having vibrations both in the plane of incidence as well as perpendicular to the plane of incidence.

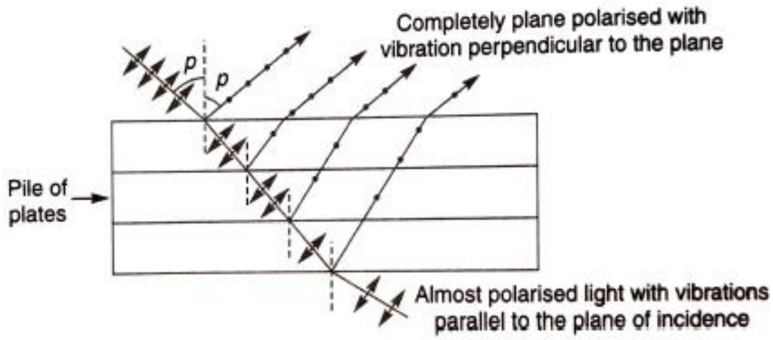


Fig. 3.9

In order to make the refracted light to be completely plane polarised light, a pile of glass plates is used. A pile of plates consists of about 9 or 10 glass plates arranged one above the other as shown in fig. 3.9.

The ordinary light is incident on pile of plates at polarising angle, a few vibrations perpendicular to the plane of incidence are reflected by the first plate and rest are refracted through it. When this beam of light is reflected by the 2nd plate, again some vibrations perpendicular to the plane of incidence are reflected by this plate and rest are transmitted.

It is important to mention here that each time, the light is incident on the surface of glass plate at Brewsters angle. The above procedure continues for different glass plates. At the last plate we get almost plane polarised light with vibrations parallel to the plane of incidence. So, the reflected light is plane polarised light.

3.4.5 Scattering of light and polarisation by scattering:

When a light wave travelling in space, it strikes an extremely small particle (compared to the wavelength of light) such as dust particle, water particle or molecules of a substance. Now a portion of the light is scattered by the particle. When the light passes through a number of particles, its intensity goes on decreasing due to scattering.

According to the Lord Rayleigh, the intensity of scattered light is:

1. Proportional to the intensity of incident light.
2. Proportional to the square of the volume of scattered particles, and
3. Inversely proportional to the fourth power of the wavelength of light used. i.e., $I \propto (1/\lambda^4)$

It has been observed that the scattered light is polarised fully or partially depending on the size of scattering particle. When the size of the particle is sufficient small, the scattered light is fully polarised while when the particles are larger, the scattered light is partially polarised.

3.6 MALUS LAW:

According to the Malus, when a completely plane polarised light beam is incident on the analyser, the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and plane of the polarizer.

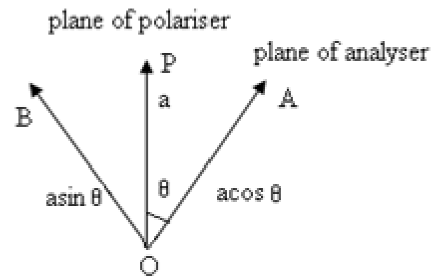


Fig. 3.10

This law holds good for the combination of the reflecting surfaces, polarising and analysing tourmaline crystals, Nicol prisms etc., but fails when the light is not completely plane polarised.

To prove this, let OP = a be the amplitude of the incident plane polarised light from the polarizer and θ, the angle between the planes of polarizer and analyser. The amplitude of the incident plane polarised light can be resolved in two components, one parallel to the plane of transmission of analyser (a cos θ) and the oter perpendicular to it (*a* sin θ). The component a cosθ is transmitted through the analyser.

Therefore Intensity of the transmitted light through the analyser

$$I_\theta = (a\ cos\ \theta)^2 = a^2 cos^2\theta$$

intensity ∝ (amplitude)^2

If I be the intensity of incident polarised light then,

$$I = a^2$$

$$I_\theta = I\ cos^2\theta$$

$$I_\theta \propto cos^2\theta$$

When θ = 0 i.e., the two planes are parallel so I_θ =1 as cos θ =1

When θ = π/2 i.e., the two planes are perpendicular, $I_\theta = 0$

The above results are experimentally observed in case of two tourmaline crystals.

3.7 Geometry of Calcite Crystal:

Calcite crystal is a colourless transparent crystal. Chemically it is hydrated calcium carbonate ($CaCO_3$). It was at one time found in great qualities in Iceland and is very large crystals of waterly clearness. Hence, it is also known as Iceland spar. It belongs to the rhombo-hedral class of hexagonal system. The 6 faces of rhombohedron are parallelograms each having angles of 101° 55′ and 78° 5′ as shown in fig. 3.11.

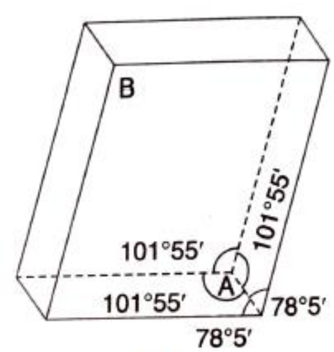


Fig. 3.11

There are two opposite corners A and B where the three obtuse angle (101°55′) meet. The corners are known as blunt corners. At the rest of six corners there is one obtuse angle and two acute angles.

1. Optic Axis:

A line passing through any of the blunt corners A and B and making equal angles with the three faces which meet, at this corner, locate the direction of the optic axis of the crystal. It may be emphasized here that optic axis is a direction and not a particular line. Hence, an optic axis can be drawn through every point in the crystal i.e., any line parallel to the line described will be optic axis.

If the rhombohedron is cut in such a way that its all edges are equal, then the line AB joining the two blunt corners or any line parallel to this will give the direction of optic axis. Crystals having one optic axis are called uniaxial crystals and those having two optics axes are called biaxial crystals (like mica).

2. Principal Section:

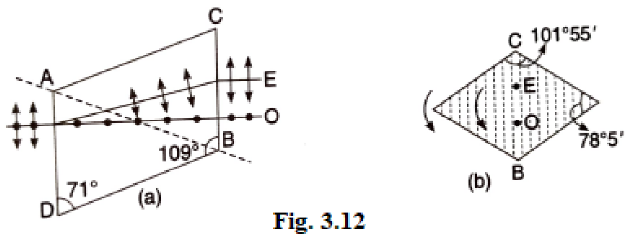


Fig. 3.12

Any plane which contains the optic axis and is perpendicular to two opposite faces is called a principal section. As a crystal has 6 faces, so for every point inside the crystal there are three principal sections, one for each pair of opposite crystal faces. A principal section cuts the crystal surfaces in a parallelogram having angle 71° and 109°. In fig the principal sections of the crystal is shown. An end on view of any principal section is a straight line shown dotted in fig in the crystal surface parallel to its shorter diagonal CB, which is the end on view of the principal sections through the blunt edges.

3.8 DOUBLE REFRACTION OR BIREFRINGENCE:

The physical properties of refraction in isotropic medium are the same in all directions but in anisotropic substances (particularly crystalline substance except those having cubic symmetry) the physical properties are different in different directions. Crystals of calcite, quartz and tourmaline are well known examples of an isotropic materials.

In 1869, Erasmus Bartholinus discovered that when a beam of ordinary unpolarised light is passed through a calcite crystal, the refracted light is split up into two rays. The one which always obey the ordinary laws of refraction and having vibrations perpendicular to the principal section is known as ordinary ray. The other, in general, does not obey the laws of refraction and having the vibrations in the principal section is known as extraordinary ray. Both the rays are plane polarised. This phenomenon is known as double refraction. The crystals showing this phenomenon are known as doubly refracting crystals.

The phenomenon of splitting of a light ray into rays when it propagates through homogenous transparent anisotropic medium is called double refraction.

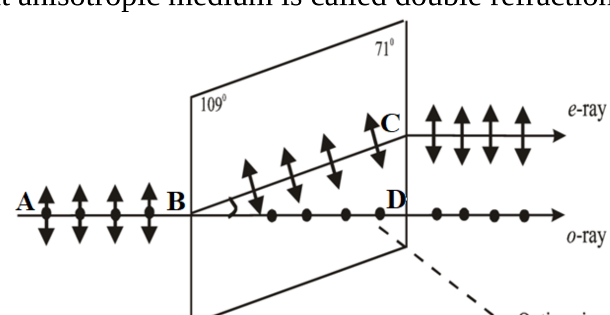


Fig. 3.13

Consider a beam AB of unpolarised light incident on the calcite crystal at an angle of incident I as shown in fig. Inside the crystal the ray breaks up into orinary and extraordinary rays. The ordinary ray travelling along BD makes an angle of refraction r_1 while the extra travelling along BC makes an angle of refraction r_2. Since, the two opposite faces of the crystals are always parallel, both the rays emerge parallel to the incident ray. The refractive index of the ordinary and extraordinary rays can be expressed as

$\mu_0 = \frac{\sin i}{\sin r_1}$ and $\mu_e = \frac{\sin i}{\sin r_2}$ respectively.

In the case of calcite $\mu_0 > \mu_e$ because $r_1 < r_2$. Therefore, the velocity of light for ordinary ray inside the crystal will be less than the extraordinary ray. It is observed that the μ_0 is same for all the angles of incidence while μ_e varies with angles of incidence. Therefore, the ordinary ray travels with the same speed in all the directions while extraordinary ray has different speed in different directions.

There are two types doubly refracting crystals: 1. Uniaxial and 2. Biaxial.

In uniaxial crystals there is only one direction along which the two refracted rays travel with the same velocity (examples are calcite, tourmaline and quartz). In biaxial crystals, there are two such directions along which the velocities are the same (examples are topaz, aragonite etc).

The phenomenon of double refraction can be illustrated with the following simple experiment. An ink dot is made on a white paper, and a calcite crystal is placed over it. Now, looking through the top face, two images are observed. If now the crystal is rotated slowly, one image remains stationary while the other rotates in the direction of rotation of crystal. The stationary image is known as ordinary image while the rotating image is known as extraordinary image.

3.9 NICOL PRISM:

Principle:

When an unpolarized light is transmitted through a calcite crystal, it splits into two beams namely o-ray and e-ray. These beams are completely plane polarized with vibrations perpendicular to each other. If by some means one beam is eliminated then the emergent beam from calcite crystal will be plane polarized light. This is achieved by using a Nicol prism.

Construction:

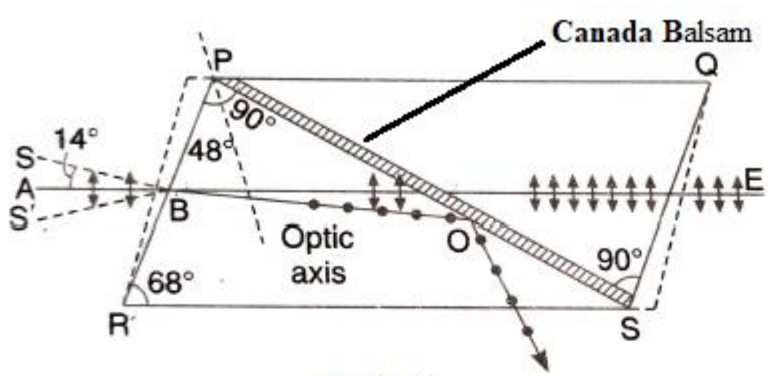


Fig. 3.14

Consider a calcite crystal whose length is three times as that of its width. The end faces of the crystal having an angle 71˚ and 109˚ with the principle section. The calcite crystal is cut into two pieces along the plane (PS) perpendicular to the principle section and as well as the end faces

PR and QS of the crystal. The end faces of the crystal are grounded in such a way that the angle in the principle section becomes 68˚ and 112˚ instead of 71˚ and 109˚. This is done to increase the field of view. The two cut pieces are cemented together by Canada balsam. Canada balsam is a transparent substance and it is optically more dense e-ray and less dense than o-ray. That means the refractive index Canada balsam is lies between the refractive indices of o-ray and e-ray. For sodium light $\mu_0 = 1.6584$, $\mu_{CB} = 1.55$ and $\mu_e = 1.4864$.

Working:

When a beam of unpolarized light enters into Nicol prism, it is doubly refracted into ordinary plane polarized light and extra ordinary plane polarized light. From the values of refractive indices, it is clear that Canada balsam acts as a rarer medium for an o-ray and denser medium for an e-ray. Therefore, there exists a critical angle of refraction for the o-ray at the interface of calcite crystal and Canada balsam surfaces but not for the e-ray. Under these conditions, if angle of incidence of the o-ray at Canada balsam greater than critical angle 69˚, it gets total internal reflection. The extra-ordinary ray is not totally reflected because it is traveling from a rarer to denser medium. Thus, only extra-ordinary ray is transmitted. Since e-ray is plane polarized having vibrations parallel to principle plane, the light emerging from the Nicol's prism is plane polarized.

Uses:

Two Nicol's prisms lined up one behind the other are often used in optical microscopes for studying optical properties of the crystal. The first Nicol, which is used to produce the plane polarized light, is called the polarizer and the second Nicol, which is used to test the light, is called the analyzer.

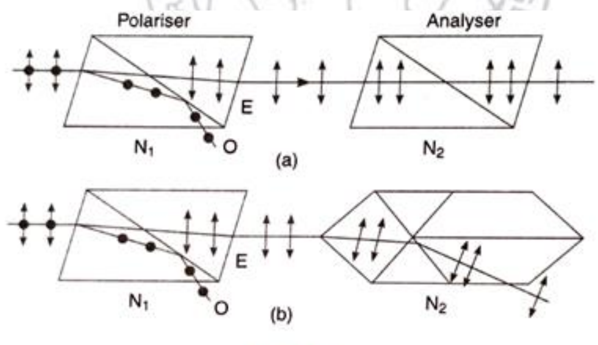


Fig. 3.15

3.10. QUARTER WAVE AND HALF WAVE PLATES

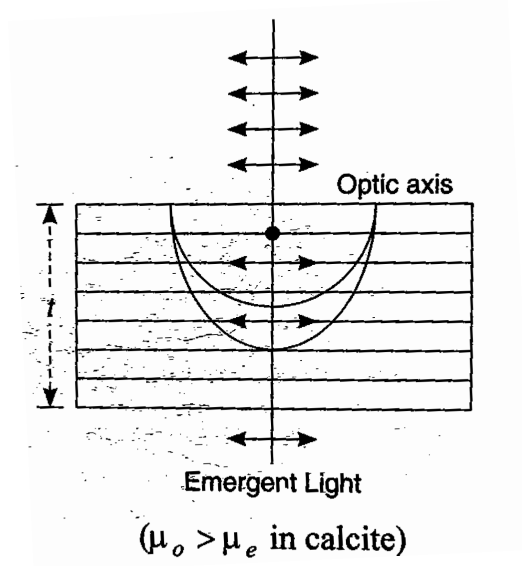
When a plane polarized light of wavelength λ is incident normally on a thin plate of uniaxial crystal cut parallel to its optic axis, the light splits up into ordinary and extraordinary plane polarized lights. They propagates along the same direction but different velocities. In positive crystals like quartz, the o-ray travels faster than e-ray. The e-ray is travels faster than o-ray in negative crystals like calcite, because the refractive index of e-ray is less compared to that of o-ray. As a result phase difference as well as path difference is introduced between them when they emerged out from other face. Hence the path difference between the two rays is

$$\Delta = (\mu_e \sim \mu_o)t$$

Two types of obstruction plates are used in optical instruments.

1. Quarter wave Plate
2. Half wave Plate

3.10.1. Quarter wave plate:



If the thickness of the crystal is such that it introduces a phase difference of $\pi/2$ or a path difference of $\lambda/4$, then the light emerging from this crystal is circularly polarized. Such a crystal is called Quarter wave plate.

$$\Delta = (\mu_o \sim \mu_e)\, t = \frac{\lambda}{4}$$

Thickness of Quarter wave plate, $t = \dfrac{\lambda}{4(\mu_o \sim \mu_e)}$

Where μ_o and μ_e are refractive indices of the crystal for ordinary and extra ordinary waves.

Uses of quarter wave plate

If linearly polarized light is incident on a quarter-wave plate at 45° to the optic axis, then the light is divided into two equal electric field components. One of these is retarded by a quarter wavelength. This produces circularly polarized light.

If circularly polarized light is incident on quarter wave plate at 45° to the optic axis then it produces linearly polarized light.

If linearly polarized light is incident on quarter wave plate other than 45° to the optic axis then it produces elliptical polarized light.

3.10.2. Half wave plate:

If the thickness of the crystal is such that it introduces a phase difference of π or a path difference of $\lambda/2$, then the light emerging from this crystal is circularly polarized. Such a crystal is called Half wave plate.

$$\Delta = (\mu_o \sim \mu_e)\, t = \frac{\lambda}{2}$$

Thickness of Quarter wave plate, $t = \dfrac{\lambda}{2(\mu_o \sim \mu_e)}$

Where μ_o and μ_e are refractive indices of the crystal for ordinary and extra ordinary waves.

Note: When the phase difference is 0, π, 2π, 3π, 4π, ... or path difference is 0, $\lambda/2$, $2\lambda/2$, $3\lambda/2$, …. the resultant light is a linearly polarized light.

Use of half wave plate

Half wave retarders can rotate the polarization of linearly polarized light to twice the angle between the optic axis and the plane of polarization. Placing the optic axis of a half wave retarder at 45° to the polarization plane results in a polarization rotation of 90° to its original plane.

3.11 PRODUCTION OF CIRCULARLY AND ELLIPTICALLY POLARISED LIGHT

(1) Circularly Polarized Light

Circularly polarized light is the resultant of two waves of equal amplitudes, vibrating at right angles to each other and having a phase difference of $\pi/2$. Following (Fig. 3.16) is the experimental arrangement to obtain circularly polarized light.

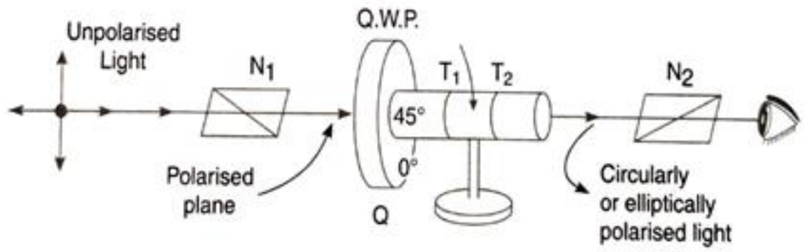


Fig. 3.16

A beam of monochromatic light is allowed to fall on a Nicol prism N_1. The emergent light from Nicol N_1 is plane polarized light. Another Nicol prism N_2 is placed at a certain distance in a crossed position, i.e., no light transmits from it. The field of view will be dark when observed from the eye in this position.

Now, a quarter wave plate Q mounted on a tube T_1 which is introduced between the two Nicols and hold normal to the incident beam. The tube T_1 can rotate about the outer fixed tube T_2. Thus, the $\lambda/4$ plate can be rotated about a horizontal axis through any desired angle. By the introduction of $\lambda/4$ plate between N_1 and N_2 the field of view after N_2 is not dark, i.e., there is some light.

The quarter wave plate is now rotated till the field of view is again dark. This happens when vibrations of light incident on quarter wave plate are along the optic axis and so perpendicular to N_2. Now, the quarter wave plate is rotated through 45° so that the vibrations of light incident on it makes angle 45° with its optic axis. At this position the amplitude of ordinary and extraordinary wave becomes equal.

According to the property of $\lambda/4$ plate a phase difference of $\pi/2$ is introduced between ordinary and extraordinary rays so that resultant beam after quarter-wave plate will be circularly polarised light.

(2) Elliptically polarised light

Elliptically polarised light is the resultant of two waves of unequal amplitudes vibrating at right angles to each other and having a phase difference of $\pi/2$. To obtain the elliptically polarised light, the experimental arrangement is the same as shown in fig. 3.16. A parallel beam of monochromatic light is allowed to fall on two Nicols in crossed position. In this case the field of view is dark. A $\lambda/4$ plate is now introduced between the two Nicols so that the field of view may be bright. The quarter wave plate is rotated in such a way that the field of view is again dark. Again the quarter wave plate is rotated such that vibration of light incident on it makes any angle other than 45°. This makes the amplitudes of ordinary and extraordinary rays unequal and so the resulting light from quater-wave plate is elliptically polarised.

3.12 CONVERSION OF ELLIPTICALLY POLARISED LIGHT INTO CIRCULARLY POLARISED LIGHT

We know that elliptically polarised light is made up of two polarised lights of unequal amplitudes at right angles to each other and having a phase difference of $\pi/2$. When a quarter-wave plate is placed in the path of this light such that its optic axis is parallel to either the major or minor axis of elliptically polarised. So further phase difference of $\pi/2$ is introduced.

Hence, the light emerging from quarter-wave plate becomes plane polarised light. In this way, the elliptically polarised light first of all is converted into plane polarised light. Again this plane polarised light is allowed to fall on a quarter-wave plate such that the plane of vibration of plane polarised light makes an angle 45° with the optic axis. The quarter-wave plate breaks the incident light (plane polarised light) into ordinary and extraordinary waves of amplitude and introduces a phase change of $\pi/2$. Thus, the emergent light is circularly polarised light.

3.13 DETECTION OF PLANE, CIRCULARLY AND ELLIPTICALLY POLARISED LIGHT

i) Plane Polarised Light

The light beam is allowed to fall on Nicol prism. If on rotation of Nicol prism, intensity of emitted light can be completely extinguished at two places in each rotation, then light is plane polarised.

ii) Circularly Polarised Light

The light beam is allowed to fall on a Nicol prism. If on rotation of Nicol prism the intensity of emitted light remains same, then light is either circularly polarised or unpolarised.

To differentiate between unpolarised and circularly polarised light, the light is first passed through quarter wave plate and then through Nicol prism. Because if beam is circularly polarised then after passing through quarter wave-plate an extra difference of $\lambda/4$ is introduced between ordinary and extraordinary component and gets converted into plane polarised.

Thus on rotating the Nicol, the light can.be extinguished at two plates.

If, on the other hand, the beam is unpolarised, it remains unpolarised after passing through quarter wave plate and on rotating the Nicol, there is no change in intensity of emitted light.

iii) Elliptically Polarised Light.

The light beam is allowed to fall on Nicol prism. If on rotation of Nicol prism the intensity of emitted light varies from maximum to minimum, then light is either elliptically polarised or a mixture of plane polarized and unpolarised.

To differentiate between the two, the light is first passed through quarter wave plate and then through Nicol prism. Because, if beam is elliptically polarised, then after passing through quarter wave plate, an extra path difference of $\lambda/4$ is introduced between 0-ray and E-ray and get converted into plane polarized. Thus, on rotating the Nicol, the light can be extinguished at two places.

On the other hand, if beam is mixture of polarised and unpolarised it remains mixture after passing through quarter wave plate and on rotating the Nicol intensity of emitted light varies from maximum to minimum.

3.14 OPTICAL ACTIVITY

The ability of substance (crystal or solution) to rotate plane of polarisation about the direction of light is called as optical activity. The substance (crystal or solution) which can rotate plane polarised light is called as optical active substance.

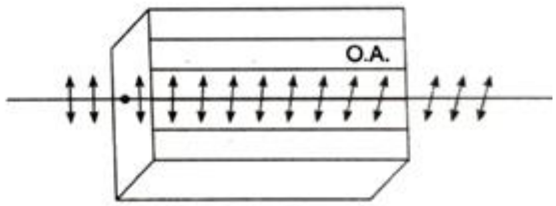


Fig. 3.18

Working:

Two Nicols are set in a crossed position. The field of view is dark. Now, a quartz plate cut with its faces parallel to the optic axis is introduced in between two Nicols, the field of view is not dark. But by slightly rotating the Second Nicol N_2 (i.e., for some different position of N_1), the field of a view is again dark. This suggests that the light emerging from quartz is still plane polarised but the plane of polarisation has rotated through certain angle. This property of rotating the plane of vibration of plane polarised light about its direction of travel by some crystal is known as optical activity.

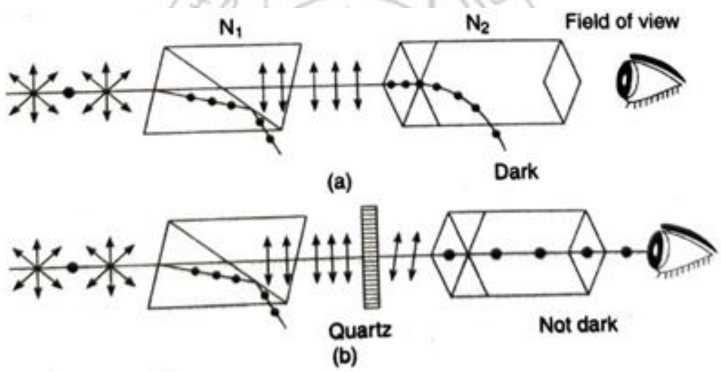


Fig. 3.19

There are two types of optically active substance

1) Dextro rotatory

The substance which rotates the plane of vibration in the clockwise with respect to the observer looking towards the source from the analyser is called as Dextro rotatory (right handed). Ex., Fruit Sugar, quartz crystal, etc.

2) Laevo Rotatory

The substance which rotates the plane of vibration in the anticlockwise with respect to the observer looking towards the source from the analyser is called as Laevo rotatory (left handed). Ex. Cane sugar solution

The amount of the angle of rotation depends on the phase difference between two circularly polarised beams in optical active substance.

Biot observed the following facts about the optical rotation:

(i) The amount of rotation θ produced by an optically active substance is proportional to its thickness (l) traversed, i.e., $\theta \propto l$.

(ii) In case of solutions and vapours, the amount of rotation for a given path length is proportional to the concentration (C) of the solution or vapour, i.e., $\theta \propto C$

(iii) The rotation varies inversely as the square of wavelength (λ) of light employed, i.e., $\theta \propto 1/\lambda$. Thus, it is least for red and greatest for violet.

(iv) The total rotation (θ) produced by a number of optically active substances is the algebraic sum of the rotations (θ_1, θ_2, θ_3, etc.) produced by individual specimens, i.e.,

$$\theta = \theta_1 + \theta_2 + \theta_3 + \ldots$$

(the rotation in the anti-clockwise direction being taken as positive and that in the clockwise direction as negative).

3.15 SPECIFIC ROTATION:

"The specific rotation of a substance at a particular temperature and for a given wavelength of light used may be *defined as the rotation produces by one decimeter length of its solution when the concentration is 1 gm/cc*.

Thus specific rotation $S = \dfrac{\theta}{lC}$

Where l is measured in decimeters

θ is measured in degrees

C is measured in gm/cc.

Note: $S = \dfrac{10\theta}{lC}$, where l is measured in cm.

Here, the angle of rotation in degrees

1. is directly proportional to the length of the solution in decimeter (l) and
2. is directly proportional to concentration (c) of solution in gm/cc.

i.e., $\theta \;\alpha\; l$

$\alpha\; C$

$\theta \;\alpha\; l\,C$

$\theta = S l\, C$

Where 'S' is called specific rotation.

3.16 LAURENT'S HALF SHADE POLARIMETER:-

The instrument used for determination of angle of rotation of an optically active substance is called a polarimeter. When it is used for finding the concentration of sugar solution it is called saccharimeter.

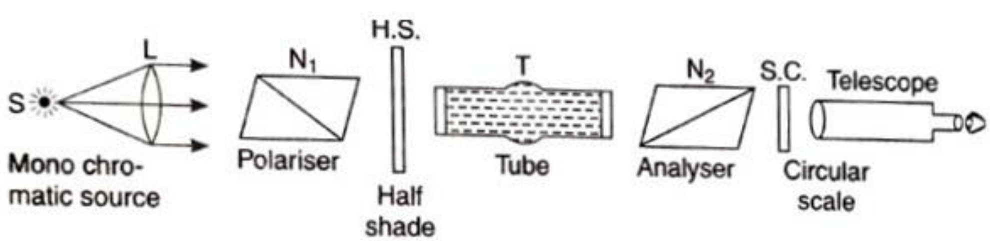


Fig. 3.20

Construction:

The essential parts of a Laurent's half-shade polarimeter are shown in fig. 3.20. The polarimeter consists of a source of light S, a convex lens L, a polariser P a half shade device, glass tube, analyser and telescope. Here hollow glass tube having a large diameter in the middle is used so that no air bubble may be in the path of light when filled with a liquid. A parallel beam of light obtained from a source S of a monochromatic light is made to fall on a polariser P, the emergent light will be plane polarised. This plane polarised light passes through a half shade device and then through the tube containing the solution of optically active substance. The emergent light passes through analyser which is viewed through a telescope.

Action of half-shade

When an optical active substance is placed in between two crossed Nicols the field of view is not dark. In order to make the field of view dark, the analyser is rotated. It is observed that when the analyser is rotated the field of view is not dark, for a considerable region. Hence, the measurement of optical rotation is not accurate.

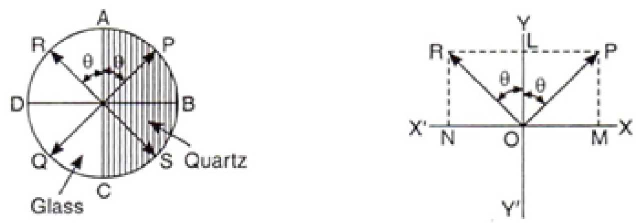
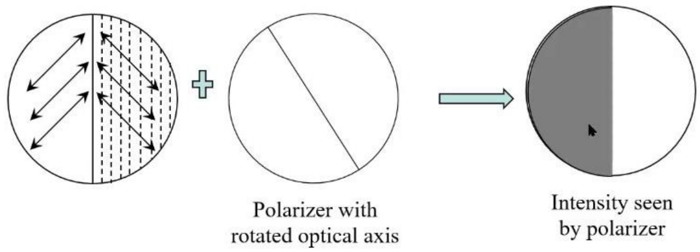


Fig. 3.21

To avoid this difficulty a half shade device is used. The Laurent's half-shade plate consists of a semi-circular half-wave plate ABC of quartz (cut parallel to optic axis) so that it introduces a phase angle of π between extraordinary and ordinary rays. A semi-circular glass plate ADC is cemented along the diameter AC. The thickness of the glass plate is such that it absorbs the same amount of light as the quartz plate. Let the plane of vibration of the plane polarised light incident normally on half-shade plate be along PO. Here, PQ makes an angle θ with AC. The vibrations emerge from the glass plate as such, i.e, along the plane PQ. Inside the quartz plate, the light is divided into two components one ordinary component along XX and the other extraordinary component parallel to optic axis, i.e., along YY axis. The two components travel along the same direction but with different speeds. The ordinary component moves with greater velocity than extraordinary component and on emergence a phase difference of t is introduced between them.

Due to this phase difference the direction of ordinary component is reversed, i.e., if the initial position of ordinary component is represented by OM [fig. 3.21], then the final position should be represented by ON. Now, the resultant of extraordinary component OL and ordinary component ON will be OR making angle θ with Y-axis. Thus, the vibration of the beam emerging out of quartz will be along RS.

If the principal plane of the analysing Nicol is parallel to PQ, then the light from glass portion will pass unobstructed while light from quartz will be partly obstructed. Due to this fact the glass half will appear brighter than the quartz half.



On the other hand, *if the principal plane of the analyser is parallel to RS, the light from quartz portion will pass unobstructed while light from glass will be partly obstructed*. Thus, the quartz half will appear brighter than the glass half If, however, the principal plane of analyser is parallel to AC (Y-axis), it is equally inclined to the two plane polarised lights and hence the field of view will be equally bright. Thus, the half shade serves the purpose of dividing the field of view in two halves. When the analysing Nicol is slightly rotated from the position of equal brightness a marked change in the intensity of two halves is observed.

Procedure:

First the tube is filled with distilled water and replace at its position. Looking through the telescope the analyser is rotated till the field of view is equally bright. The reading on the circular scale is noted as θ_1.

Now the tube is filled with given solution and replace it in this position. On looking through the telescope we find that one half of the field of view is less bright than the other. Then again the analyser is rotated till the field of view becomes equally bright. The reading on the circular scale is noted as θ_2.The difference of two readings gives an angle θ. Here θ is called as the angle of rotation.

$$\theta = \theta_1 \sim \theta_2$$

The length of the solution in the tube is measured, by knowing the value of C. we can calculate the specific rotatory power by using the formula.

$$S = \frac{10\theta}{lC}$$

Where l = length of solution in cm.

C = concentration of the solution in gm/cc.

UNIT-IV

ABERRATIONS AND FIBRE OPTICS

4.1 INTRODUCTION

The simple equation derived for a lens between object and image distances, radii of curvature, refractive index, focal lengths, etc. are based on the assumptions: (i) all the incident rays made small angles with the principal axis, (ii) the aperture of the lens is small, and (iii) a point object gives a point image.

In practice, however, lenses are used to form images of points situated off the axis also. Moreover, due to the finite size of the lens, the cone of the light rays which forms the image of the point object is of finite size.

We know that deviation produced in any ray depends on the height of its point of incidence. On the lens, different rays come to focus at different points. Thus, the image is not sharp. The refractive index and hence, the focal length of a lens are different for different wavelengths of light. When the incident light is not monochromatic, different colours are focused at different points. Thus, the lens forms a number of images of different colours at different positions.

The deviations from the size, shape and position of an image as calculated by simple equations are called aberrations produces by a lens. The defaults are mainly

1. **Due to light**: If the light is non monochromatic (white light) then the image becomes multicoloured and the defect is known as chromatic aberration.

2**. Due to Optical system**: Even with monochromatic light, several defects in shape of image are found. They are known as monochromatic aberrations. They are of 5 types.

1. Spherical aberration
2. Coma
3. Astigmatism
4. Distortion
5. Curvature

4.2 Chromatic aberration

Chromatic aberration is due to the refractive index of the material of the lens. The refractive index of the material of the lens is different for different wavelengths of light. Hence the focal length of a lens is different for different wavelengths. As the magnification of the image is dependent on the focal length of a lens, the size and position of the image is different for different wavelengths (colours).

Thus, the image of white object formed by a lens is coloured and blurred. This defect of the image is known as chromatic aberration.

Longitudinal chromatic aberration

The formation of the images of different colours in different positions along the axis is known as longitudinal or axial chromatic aberration.

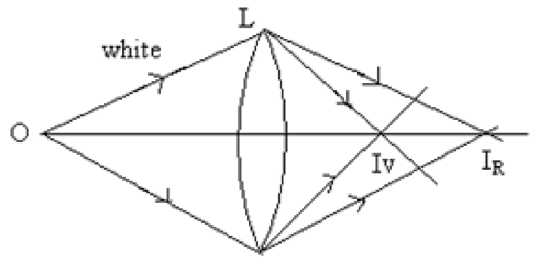


Fig. 4.1

Fig. 4.1 shows a white point object O situated on the axis of a lens. The violet and red images on the axis are shown as I_V and I_R respectively. The distance between I_V and I_R is a measure of longitudinal chromatic aberration.

Longitudinal chromatic aberration = I_R-I_V

Lateral chromatic aberration

The variation in the size of image is called as lateral chromatic aberration.

Fig. 4.2 shows a convex lens and an object AB placed in front of the lens. The lens forms the image of white objet AB as A_VB_V and B_RA_R respectively. The images of different colours are formed of different sizes. This is due to the magnification of white object is different for different colours. y gives the lateral chromatic aberration.

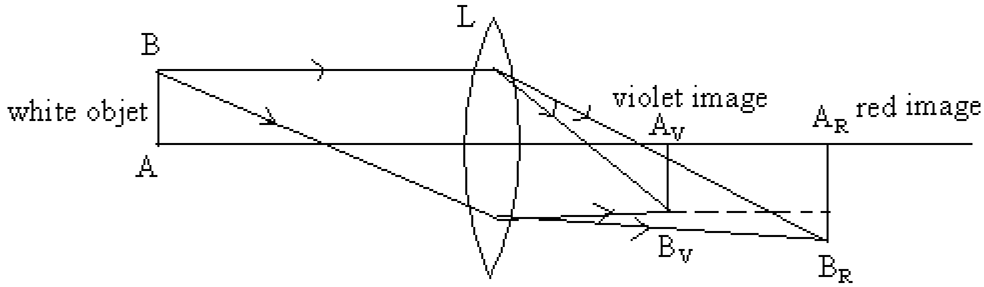


Fig. 4.2

<u>Expression for longitudinal chromatic aberration</u>:

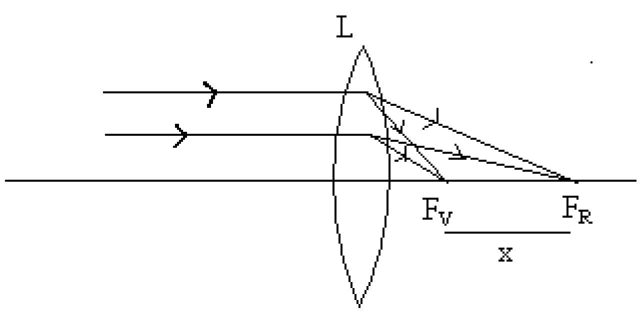


Fig. 4.3

When a parallel beam of white light is passed through a lens, the beam gets dispersed and rays of light of different colours come to focus at different points along the axis.

The violet rays of light come to focus at F_V and red rays come to focus at F_R.

The distance $x = F_R - F_V$ is called longitudinal chromatic aberration.

The focal length of the lens is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{f(\mu - 1)} \text{ ----------------------- (1)}$$

Similarly for violet $\frac{1}{f_V} = (\mu_V - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f_V} = \frac{(\mu_V - 1)}{f(\mu - 1)} \text{ ---------------- (2)}$$

Similarly for red $\frac{1}{f_R} = \frac{(\mu_R - 1)}{f(\mu - 1)}$ -------------- (3)

(2) - (3) => $\frac{1}{f_V} - \frac{1}{f_R} = \frac{(\mu_V - 1)}{f(\mu - 1)} - \frac{(\mu_R - 1)}{f(\mu - 1)}$

$$= \frac{\mu_V - 1 - (\mu_R - 1)}{f(\mu - 1)}$$

$$\frac{f_R - f_V}{f_V f_R} = \frac{\mu_V - \mu_R}{f(\mu - 1)}$$

But, $\frac{\mu_V - \mu_R}{(\mu_Y - 1)} = \omega$, called as dispersive power of material of the lens.

Here $\mu_Y = \frac{\mu_V + \mu_R}{2}$

$$\therefore \frac{f_R - f_V}{f_V\ f_R} = \frac{\omega}{f_Y}$$

But mean focal length $f_Y = \sqrt{f_V\ f_R}$

$$\therefore \frac{f_R - f_V}{{f_Y}^2} = \frac{\omega}{f_Y}$$

$$f_R - f_V = \omega f_Y$$

4.3 ACHROMATISM OF LENSES (*Achromatic Doublet*)

When the two lenses are placed in such a way that the imagtl formed by them is free from chromatic aberration. The combination of lenses is called as *achromatic combination* and the phenomenon is known as *achromatism.*

The minimization or removal of chromatic aberration is termed as achromatization.

We have seen that a convex lens and concave lens forms the image of violet colour closer to the lens than red colour. It is observed that although the violet colour is focused nearer to the lens in both the lenses but the violet colour falls to the left of red colour in case of convex lens and to the right of red colour in concave lens. Hence, it is possible that a combination of a suitable convex lens with a concave lens may be free from chromatic aberration. Such a combination is known as *achromatic doublet.*

To achieve the achromatic doublet a crown glass convex lens of low focal length and a flint glass concave lens of greater focal length are used. There are two important cases for achromatic process.

Achromatic conditions

Chromatic aberrations are eliminated by

1. Keeping two lenses in contact with each other

2. Keeping two lenses out of contact

<u>4.3.1. Condition for Achromatism of two thin lenses placed in contact (Achromatic doublet):</u>

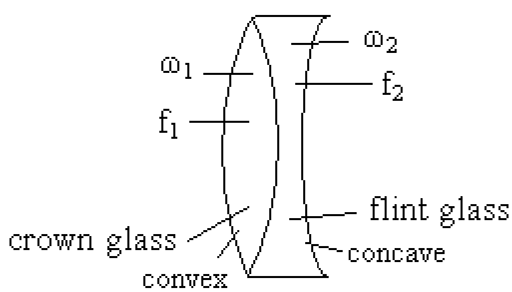


Fig. 4.4

Let f_1 and f_2 be the focal lengths of the two lenses in contact and ω_1 and ω_2 be their dispersive powers.

If F be the focal length of the above combination, then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

Differentiating the above equation we get

$$\frac{-dF}{F^2} = \frac{-df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

$$\text{But } \frac{df_1}{f_1^2} = \frac{\omega_1}{f_1}, \quad \frac{df_2}{f_2^2} = \frac{\omega_2}{f_2}$$

$$\therefore \frac{dF}{F^2} = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2}$$

For Achromatism dF = 0

$$\therefore \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

$$\frac{\omega_1}{f_1} = \frac{-\omega_2}{f_2}$$

$$\therefore \frac{\omega_1}{\omega_2} = \frac{-f_1}{f_2}$$

Since ω_1 and ω_2 are positive, f_1 and f_2 must be opposite in sign. i.e., one is convex and other should be concave. Since the achromatic doublet is to behave as convex lens f_1 must be less than f_2 and ω_1 must be less than ω_2. Generally convex lens is made of crown glass and concave lens is made of flint glass.

4.3.2. Condition for Achromatism of two thin lenses separated by a finite distance:

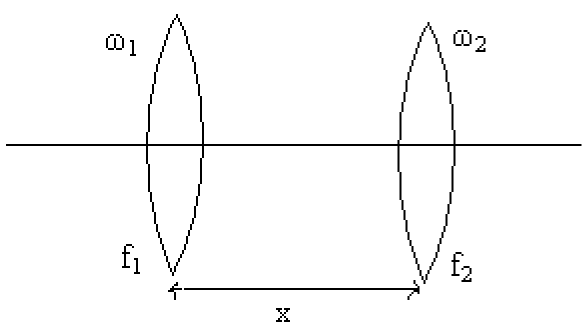


Fig. 4.5

Let us consider two convex lenses of focal lengths f_1 and f_2 separated by a distance 'x'. If f is the focal length of the above combination

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

Differentiating the above equation, we get

$$\frac{-dF}{F^2} = \frac{-df_1}{f_1^2} - \frac{df_2}{f_2^2} - x\left[\frac{-df_1}{f_1^2 f_2} - \frac{df_2}{f_1 f_2^2}\right]$$

$$\text{But } \frac{df_1}{f_1^2} = \frac{\omega_1}{f_1}, \quad \frac{df_2}{f_2^2} = \frac{\omega_2}{f_2}$$

$$\therefore \frac{dF}{F^2} = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - x\left[\frac{\omega_1}{f_1 f_2} + \frac{\omega_2}{f_1 f_2}\right]$$

For Achromatism *dF= 0*

$$\therefore \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = x\left[\frac{\omega_1}{f_1 f_2} + \frac{\omega_2}{f_1 f_2}\right]$$

$$\frac{\omega_1 f_2 + \omega_2 f_1}{f_1 f_2} = \frac{\omega_1 + \omega_2}{f_1 f_2} x$$

$$x(\omega_1 + \omega_2) = \omega_1 f_2 + \omega_2 f_1$$

$$x = \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2}$$

In general $\omega_1 = \omega_2 = \omega$

$$x\,(2\omega) = \omega f_2 + \omega f_1$$

$$x = \frac{f_1 + f_2}{2}$$

For Achromatism the distance between the two lenses must be equal to half the sum of their focal lengths.

4.4 MONOCHROMATIC ABERRTIONS:

The aberrations produced in an image even when a monochromatic light is used are called monochromatic aberrations. These aberrations are the mainly due to (i) the large aperture of the lens, (ii) the large size of the object, (iii) the large angle subtended by the rays with principle axis.

Following are the different types of monochromatic aberrations.

1.Spherical aberration
2.Coma
3.Astigmatism
4.Curvature of the field
5.Distortion

The first three produce blurring of image. The last two (curvature and distortion) cause dislocation within the image but no blur.

4.4.1. SPHERICAL ABERRATION:-

The failure or inability of the lens to form a point image of an axial point object is called spherical aberration.

Consider a point source O of monochromatic light placed on the axis of a large aperture convex lens. The rays which are incident near the axis (called paraxial rays) come to focus at point I_p while the rays incident near the rim of the lens (called marginal or peripheral rays) come to focus at I_m. The intermediate rays are brought to focus between I_m and I_p. It is clear from the figure that the paraxial rays form the image at a point at a greater distance than the marginal rays. Thus, the image is not sharp at any point on the axis.

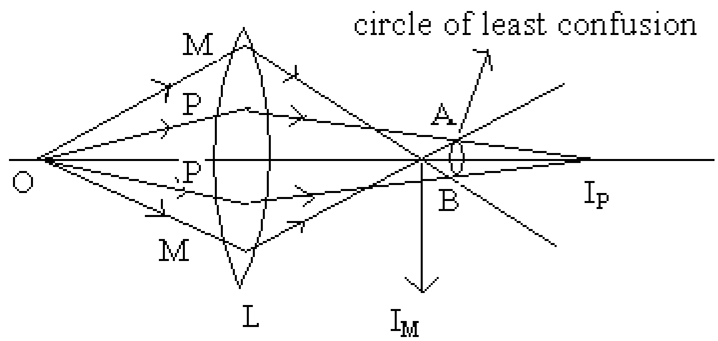


Fig. 4.6

Reason for spherical aberration

The lens can be supposed to be divided into circular zones. The focal lengths are slightly vary with the radius of the zone, i.e., different zones have different focal lengths. The focal length of marginal zone is lesser than the paraxial zone, hence the marginal rays are focused first.

The marginal rays suffer greater deviation than the paraxial rays because they are incident at a greater height than the latter. The distance between I_m and I_P is the measure of longitudinal or axial spherical aberration.

If a screen is placed normal to the principal axis at I_m or I_p then the image on the screen consists of circular disc which sharply focused at the center but diffused near the outer edge. If the screen is moved between I_m and I_p, then the size of the disc is minimum in the position AB where the paraxial and marginal rays cross. Due to the smallest cross-section, this is known as circle of least confusion. This is the nearest approach to a point image, i.e., if the screen is placed in the position AB, then we obtain best possible image of uniform sharpness. The radius of the circle of least confusion is called the lateral spherical aberration.

The spherical aberration produced by a concave lens is negative while for the convex lens it is taken as positive.

MINIMISING METHODS:

METHOD -1: By using stops:

By reducing the aperture of the lens using proper stops, the spherical aberration can be minimized.

By reducing the aperture of the lens the distance between paraxial and marginal focii reduced. Thus the spherical aberration is reduced. But drawback in this method is the intensity of the image also much reduced.

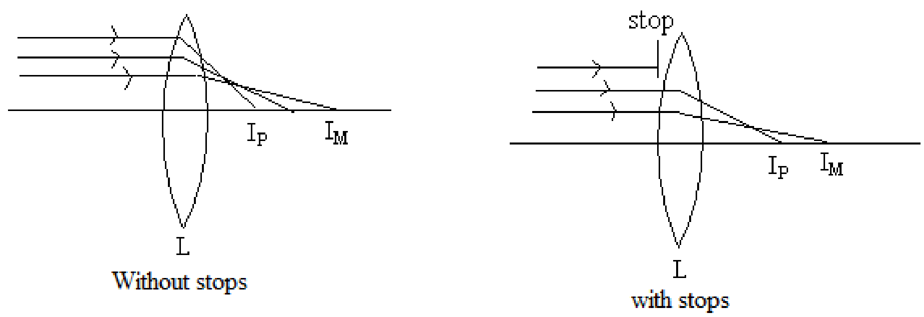


Fig. 4.7

<u>METHOD -2</u>:<u>By using Plano convex lens (or) By distributing the deviations equally at the two surfaces of a lens</u>:

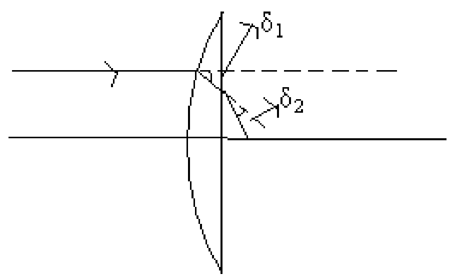


Fig. 4.8

It is experimentally found that the spherical aberration is directly proportional to the square of the total deviation produced by lens. It can be shown that this deviation is distributed equally at the two surfaces of a lens, the spherical aberration will become minimum.

<u>Proof</u>: Let δ_1 and δ_2 be the deviations at the two surfaces of a lens then the total deviation

$$\delta = \delta_1 + \delta_2$$

But spherical aberration $\alpha\ \delta^2$

$$\alpha\ (\delta_1 + \delta_2)^2$$

$$\alpha\ (\delta_1 - \delta_2)^2 + 4\delta_1\delta_2$$

If $\delta_1 = \delta_2$ then, Spherical aberration $\alpha\ 4\delta_1\delta_2$

From this we can readily understood that the spherical aberration will become minimum.

The general rule to minimize the spherical aberration in a plano-convex lens is that the convex side should face the incident beam or emergent beam whichever is more parallel to the axis.

<u>METHOD-4: By using two suitable lenses in contact</u>

It has been observed that in case of a convex lens, the marginal image *Im* lies towards the left of paraxial image I_p while in case of a concave lens, the marginal image I_m lies towards the right of paraxial image I_p. Thus, by a suitable combination of two lenses, the spherical aberration may be minimized. The difficulty with this combination is that it works only for a particular pair of object and image for which it is designed.

<u>METHOD -3</u>: <u>By using two Plano convex lenses separated by a finite distance</u>:

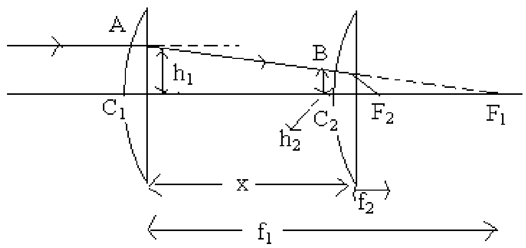


Fig. 4.9

Consider two co-axially arranged Plano convex lenses with their surfaces turned towards object. Their focal lengths are f_1 and f_2. They are separated by a distance 'x'.

A ray of light incident on the first lens at a height h_1 from the principle axis. It is refracted and incident on the second lens at a height h_2 and finally converges at F_2. Let δ_1 and δ_2 be the total deviations produced by the two lenses.

We know that the spherical aberration is minimum when $\delta_1 = \delta_2$.

But $\delta = \frac{h}{f}$

$$\therefore \frac{h_1}{f_1} = \frac{h_2}{f_2}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{f_1}{f_2} \quad (1)$$

From the figure triangles, AC_1F_1 and BC_2F_1 are similar

$$\therefore \frac{AC_1}{C_1F_1} = \frac{BC_2}{C_2F_1} \quad \Rightarrow \quad \frac{h_1}{h_2} = \frac{f_1}{f_1 - x} \quad (2)$$

From (1) & (2)

$$\frac{f_1}{f_2} = \frac{f_1}{f_1 - x}$$

$$f_2 = f_1 - x$$

$$x = f_1 - f_2$$

Hence the spherical aberration is minimum when the distance between the two lenses is equal to the difference between foal lengths of the two lenses. This condition is satisfied in the case of Huygen's eyepiece.

<u>METHOD -4</u>:<u>Crossed lens</u>:

It can be proved that the spherical aberration produced by a lens will be minimum if

$$\frac{R_1}{R_2}=\frac{2\mu^2-\mu-4}{\mu(2\mu+1)}$$

Here R_1, R_2 are the radii of curvature of the lens, μ is refractive index of the material of the lens. The lens which satisfies the above condition is called as crossed lens.

(i) If μ = 1.5, then $\frac{R_1}{R_2}=\frac{-1}{6}$

(ii) If μ = 1.5, then $\frac{R_1}{R_2}=\frac{-1}{6.5}$

In these casse the crossed lens is almost a plano-convex lens.

(iii) If μ = 1.686, then $\frac{R_1}{R_2}=0$

In this case the crossed lens is exactly a plano-convex lens. Hence, a plano-convex lens of μ = 1.686 also serves the purpose of crossed lens.

METHOD -5: By using Aplanatic lens:

A spherical lens which is free from the defects of spherical aberration and coma is called an aplanatic Lens. An aplanatic surface is characterized by the property of forming a point image of a point object placed on the principal axis.

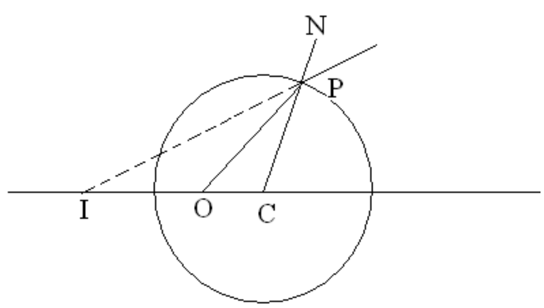


Fig. 4.10

The pair of conjugate points in the lens system free from spherical aberration and coma are called aplanatic Points.

Positions of Aplanatic points:

Let 'C' be the centre of curvature of a spherical surface of radius of curvature 'R' and refractive index 'μ'. Consider a point object 'O' placed on the axis at a distance $\frac{R}{\mu}$ from C. A ray of light OP making an angle θ_1 with the axis, after refraction it will bent away from the normal line. When produced back, it will meet the axis at I making an angle θ_2 with the axis. The angles of incidence and refraction are 'i' and 'r' respectively.

From Snell's law $\frac{1}{\mu}=\frac{\sin i}{\sin r}$

$$\mu \sin i = \sin r \text{ --------- } (1)$$

From Δle OCP,

$$\frac{OC}{\sin i} = \frac{CP}{\sin \theta_1}$$

$$\frac{R}{\mu \sin i} = \frac{R}{\sin \theta_1}$$

$$\mu \sin i = \sin\theta_1 \text{------------ } (2)$$

From (1) and (2) $\sin\theta_1 = \sin r$

$$\Rightarrow \theta_1 = r$$

From Δle IPO, $\theta_2 + r - i + 180 - \theta_1 = 180$

$$\Rightarrow \theta_2 - i = 0$$

$$\Rightarrow \theta_2 = i \text{ -------------- } (3)$$

From Δle IPC, $\tan \theta_2 = \frac{CP}{CI}$

But $\tan \theta_2 = \theta_2 \Rightarrow \theta_2 = \frac{R}{CI}$

From Δle OPC, $\tan i = \frac{OC}{CP}$

But $\tan i = i \Rightarrow i = \frac{R}{\mu R} = \frac{1}{\mu}$

From (3) $\frac{R}{CI} = \frac{1}{\mu}$

$$CI = \mu R$$

This relation does not contain θ_1 and θ_2. This shows that if the object is placed at a distance $\frac{R}{\mu}$ from C then the image is formed at a distance μR from C. Hence all the rays starting from O will appear to diverge from I.

∴ The image is free from spherical aberration. O and I are called aplanatic Points.

4.4.2. COMA:-

If the point object is situated slightly off the principle axis, the lens even corrected for spherical aberration forms an egg-like (comet like) image in the place of point image. This defect in the image is called Coma.

Consider an off axis point A in the object OA. The rays leaving A and passing through the different zones of the lens such as (1, 1), (2, 2), (3, 3), etc are brought to focus at different points B_1, B_2, B_3 etc. The radius of these circles gone increase with increasing radius of the zone. The resultant image is comet-like. So, the image is not in a perfect focus. This defect is known as coma.

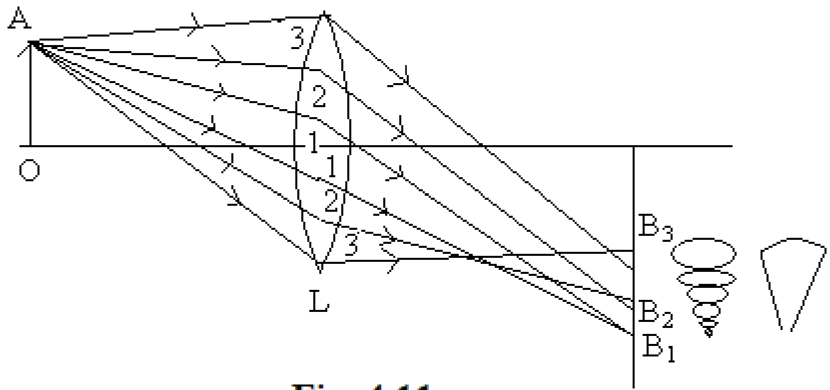


Fig. 4.11

This defect arises due to

(i) Different zones of the lens having different focal lengths and different lateral magnification. So different zones form different sized images.

(ii) Each zone forms the image of a point in the form of a circle. Let the zone have pairs of diametrically opposite points like (a, a); (b, b); (c, c); (d, d) as shown in figure. The rays starting from point A and passing through these pairs will be focused at a', b', c' and d' respectively to form a comatic circle. The radius of the comatic circle increases as the radius of the zone of the lens increases. Thus, the resultant image is comet like.

REDUCING METHODS:

1. Coma may be reduced to a certain extent by the use of proper stop placed at a suitable distance from the lens. This restricts the outer zones and allows only the central zones to refract the rays.

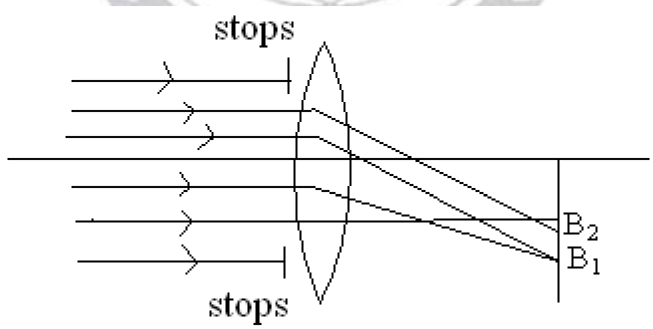


Fig. 4.12

2. Coma may be reduced using crossed lens of μ=1.5, $\frac{R1}{R2} = -\frac{1}{9}$. If the lens are designed in such a way we can reduce the coma effectively.

3. Abbe showed that lenses are completely free from spherical aberration and coma if the following sine condition is satisfied:

$$\mu_1\, y_1 \sin\theta_1 = \mu_2\, y_2 \sin\theta_2$$

Here θ_1 and θ_2 are the angles which the conjugate rays make with the axis, μ_1 and μ_2 are the refractive indices of the object and image spaces and y_1 and y_2 are the lengths of the object and of the image for the zone. When the medium on both sides is the same, then the condition reduces to

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{y_1}{y_2}$$

If the lateral magnification is the same for all the rays of light, irrespective of the angles then coma may be eliminated. A lens satisfying the condition ($\sin\theta_1/\sin\theta_2$) = constant, is called an aplanatic lens.

ABBE SINE CONDITION

Consider a spherical refracting surface of radius of curvature 'R' as shown in the figure. Let μ_1 and μ_2 be the refractive indices of the media on the left and right of the surfaces respectively. PP^1 is the object of height y_1 placed at a distance u from the pole 'O' of the surface $I\,I^1$ is the image of the object of height y_2 formed at a distance ν from the pole O of the surface. From the figure i and r are the angles of incidence and refraction.

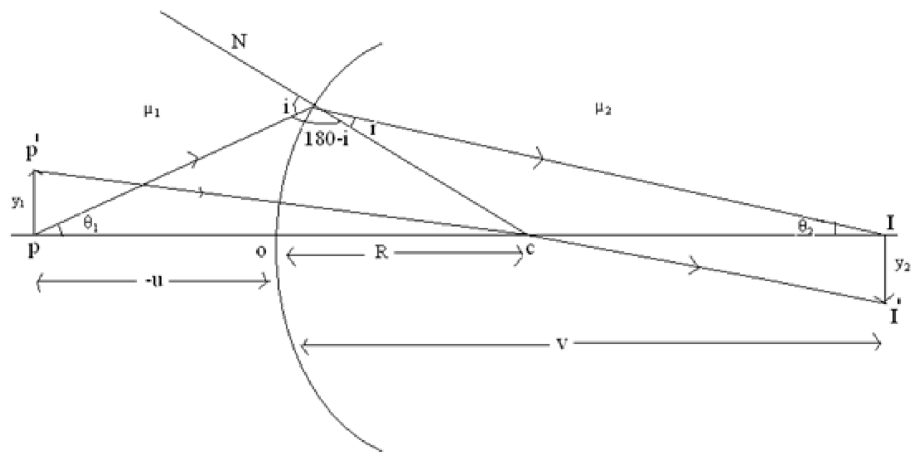


Fig. 4.13

From triangle PCP^1 and ICI^1, $\frac{P\,P^1}{I\,I^1} = \frac{P\,C}{I\,C}$

$$\Rightarrow \quad \frac{P\,P^1}{P\,C} = \frac{I\,I^1}{I\,C}$$

$$\Rightarrow \quad \frac{y_1}{R - u} = \frac{y_2}{\nu - R}$$

$$\Rightarrow \quad \frac{y_1}{y_2} = \frac{R - u}{\nu - R} \qquad \text{----------} (1)$$

From Snell's law, $\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$ ---------- (2)

From triangle PNC, $\frac{\sin\theta_1}{CN} = \frac{\sin(180 - i)}{CP}$

$$\frac{\sin\theta_1}{R} = \frac{\sin i}{R-u} \qquad \text{---------- (3)}$$

From triangle, $\dfrac{\sin\theta_2}{CN} = \dfrac{\sin r}{CI}$

$$\frac{\sin\theta_2}{R} = \frac{\sin r}{v-R} \quad => \quad \frac{\sin\theta_2}{\sin r} = \frac{R}{v-R} \qquad \text{------ (4)}$$

$$\frac{\sin\theta_2}{\sin r}\frac{\sin i}{\sin\theta_1} = \frac{R}{v-R}\frac{R-u}{R}$$

$$\Rightarrow \quad \frac{\sin\theta_2}{\sin\theta_1}\frac{\sin i}{\sin r} = \frac{R-u}{v-R}$$

$$\Rightarrow \quad \frac{\sin\theta_2}{\sin\theta_1}\frac{\mu_2}{\mu_1} = \frac{y_1}{y_2}$$

$$\Rightarrow \quad \mu_1\, y_1 \sin\theta_1 = \mu_2\, y_2 \sin\theta_2$$

This condition is known as ABBE'S sine condition. The lens which satisfies the above condition is free from Coma.

4.4.5 ASTIGMATISM:

When a point object is situated far off the axis of a lens, the image formed by the lens is not in a perfect focus. The image consists of two mutually perpendicular lines separated by a finite distance. Moreover, the two lines lie in perpendicular planes. This defect of the image is called as **astigmatism**. The astigmatism is shown in figure.

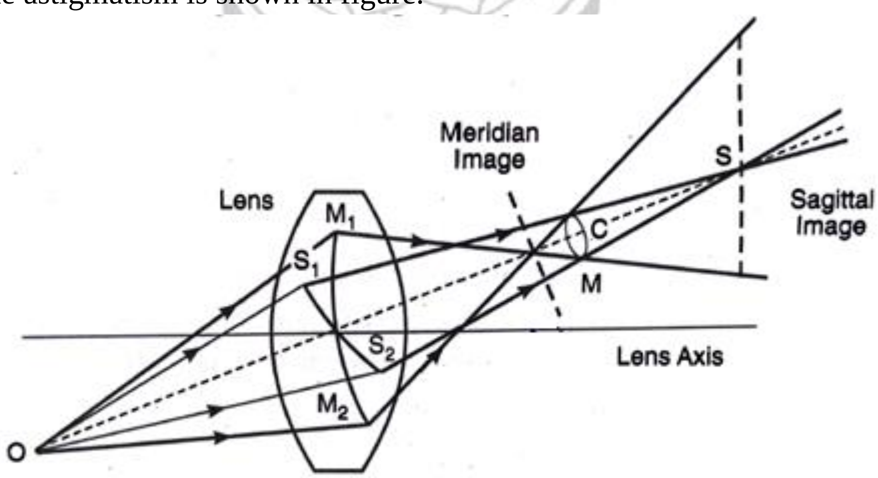


Fig. 4.14

O is a point object situated far off the axis of the lens. $\mathbf{M_1M_2}$ is the meridian plane of the lens, i.e., a plane containing O and the principal axis of the lens. $\mathbf{S_1S_2}$ is the sagittal plane of the lens, i.e., the plane perpendicular to meridian plane and passing through the principal axis. Now, consider the rays from O and passing through the two planes $\mathbf{M_1M_2}$ and $\mathbf{S_1S_2}$. The rays through the meridian plane after refraction come to a horizontal line focus **M** while the rays through the sagittal plane come to a focus farther away from the lens in a vertical line focus **S**. So, if the screen is moved between **M** and **S**, an irregular patch of light is obtained. Thus, the image never approaches to a point image. The defect is known as astigmatism. When the screen is moved between S and M, the patch of the light reduces to a circle as shown by C in figure. This is known as the *circle of least confusion.* This is the nearest approach to a point image. The difference between the lines (M and S) is a measure of astigmatism and is called the astigmatic difference.

Removal of astigmatism

The defect of astigmatism may be eliminated by the following methods:

(i) The astigmatism is due to large inclination of the rays with the axis of the lens. Therefore, if the rays making large angles with the axis are cut off, the defect can be eliminated. This is done with the help of stop by placing it in a suitable position as shown in figure. Now, only less oblique rays are permitted to form the image.

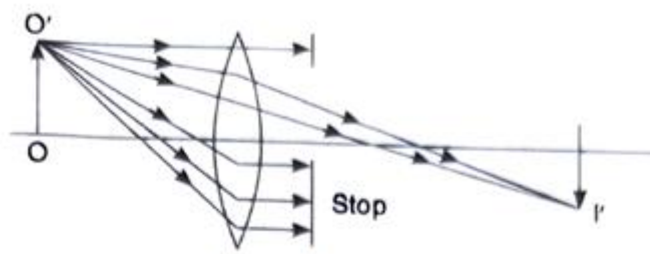


Fig. 4.15

(ii) In case of a concave lens, the astigmatic difference is negative. Therefore, a suitable convex and concave lens separated at a suitable distance may be used to reduce the astigmatism.

(iii) For a system of several lenses, the astigmatism may be eliminated by adjusting their positions. Such systems are widely used as photographic objectives on which narrow pencils of rays are incident at large angles.

4.4.6 CURVATURE

Even when the lens is free from spherical aberration (coma and astigmatism) the image of an extended plane object OO' is curved as shown in figure. If a screen is placed at I perpendicular to the axis of lens, then the image II' will not be in focus but this is curved. This defect is called curvature. This arises due to the fact that points away from the axis, such as O', are at a greater distance from the center C Of the lens than the axial point O. Therefore, the image I' formed at a smaller distance than I.

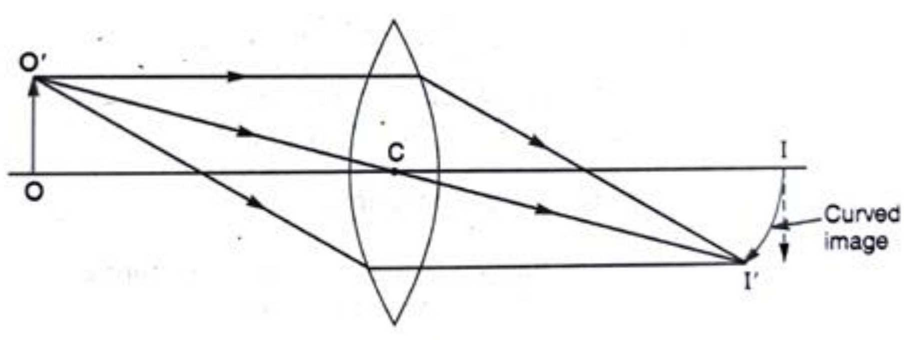


Fig. 4.16

Removal of Curvature

The curvature may be removed by the following methods:

1. For a single lens, the curvature may be reduced by placing an aperture in a suitable position in front of the lens.

2. For a combination of lenses, the condition for absence of curvature is

$$\Sigma \frac{1}{\mu f} = 0$$

In case of two lenses, $\mu_1 f_1 + \mu_2 f_2 = 0,$

Here $\mu_1$and μ_2 are refractive indices and f_1 and f_2 are the focal lengths of the lenses.

Here $\mu_1$and μ_2 are both positive so that f_1 and f_2 must necessarily be of opposite signs, i.e., if on lens is convex then the other lens should be concave.

The above condition holds good whether the lenses are in contact or they are separated at a certain distance.

4.4.7. DISTORTION:

The image of a plane square like object placed perpendicular to the axis of the lens is not of the same shape, i.e., it is distorted. This defect is known as distortion. This defect arises due to the fact the same lens produces different magnification for different axial distances. Therefore, different parts of the object suffer from different magnifications.

The distortion is of the two types

1. Barrel shaped distortion. 2. Pin-cushion distortion.

1. Barrel shaped distortion:

When the real image of a rectangular piece of wire gauge [fig. 4.17 (a)] formed on the screen by a convex lens appears like a barrel shaped [fig. 4.17 (b)], it is known as “barrel shaped distortion”.

(a) No distortion (b) Barrel distortion (c) Pin cushion distortion

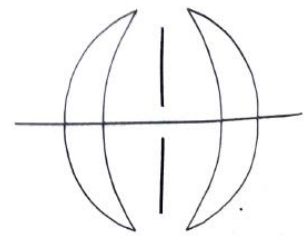
Fig. 4.17

2. Pin-cushion distortion:

When the image of a rectangular object formed by a convex lens is like shown in fig. (21c), the defect is known as pin-cushion distortion. In this case, the magnification increases with increasing axial distance from the center.

Elimination of distortion:

1. This defect can be minimized by using a thin lens.
2. The distortion can be removed by using a combination of two similar meniscus convex lenses, with their concave surfaces facing each other and placing an aperture stop in the middle. Such a combination is called as an *orthoscopic doublet* or a *rapid rectilinear lens*.



FIBRE OPTICS

4.5 INTRODUCTION

Basically the communication system consists of three parts (i) transmitter, (ii) transmission channel (may be either a guided transmission line such a wire or waveguide) and (iii) receiver. Using a transmission line, the signal gets progressively attenuated and distorted when carrying data over long distances. So the improvement in the communication process needed in terms of improving the transmission methods as well as data rate of transmission. After the development of lasers, reliable and powerful coherent radiation became available. Light used for communication purposes have an advantage over conventional radio and micro-waves due to more information carrying capacity. But light energy rapidly dissipated in the open atmosphere. So a certain medium is necessary like metal wires for guiding electrical current. As a reason Optical Fibres are developed.

Optical fiber lines offer several important advantages over wire lines. Optical fibers are the light equivalent of microwave guides with the advantage of very high bandwidth and high information carrying capacity. Optical fibre provides the necessary wave guide for light. Fibre optics is a technology related to transportation of optical energy (light energy) in guiding media specifically glass fibres. The guiding medium to light is called optical fibre. The communication through optical fibre is called as light wave communication or optical communication.

Optical fiber technology is increasingly replacing wire transmission lines in communication systems and is expected to be as common as electrical wiring even in our vehicles and houses very shortly. Earlier, the fiber optics communication systems are very expansive than equivalent wire or radio-systems. Now the situation is changed very much, optical fiber systems have become comparative with other systems in price and eventually started replacing them.

4.6 OPTICAL FIBRE AND ITS WORKING PRINCIPLE

An optical fibre is a hair-thin cylindrical fibre of glass or any transparent dielectric medium. In practical applications, they consist of many thousands of very long fine quality glass/quartz fibres. The optical fibres are based on the principle of total internal reflection. The fibres are coated with a layer of lower refractive index. When light is incident on one end of the fibre at small angle, it passes through the fibre as below.

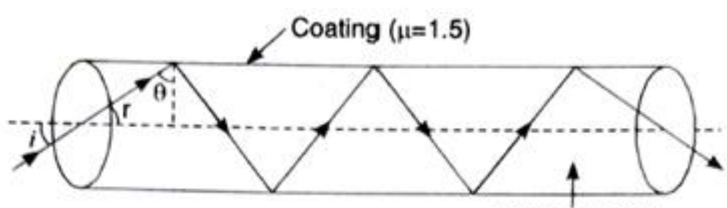


Fig. 4.18

Let i be the angle of incidence of the light ray with the axis and r, the angle of refraction. If θ be the angle at which the ray is incident on the fibre boundary, then θ = (90°- r). Suppose n_1 be the refractive index of fibre. If $\theta \geq \theta_c$ critical angle where $\theta_c = \sin^{-1}(1/n_1)$ then the ray is totally internally reflected. In this way the ray undergoes repeated total internal reflections until it emerges out of the end of the fibre, even if the fibre is bent. Thus, the light ray is guided through the fibre from one end other end without any energy being lost due to refraction.

In case of optical fibres, it is essential that there must be very litle absorption of light as it travels through a long distance inside optical fibre. This is achieved by purification and special preparation of the material used.

4.7 STRUCTURE OF OPTICAL FIBRES

The fibres which are used for optical communication are wave guides made of transparent dielectrics. Its function is to guide visible and infrared light over long distances. An optical fibre consists of an inner cylinder which is made of glass, called the core. The core carriers light. The core is surrounded by another cylindrical shell of lower refractive index called the cladding. The cladding helps to keep the light within the core through the phenomenon of total internal reflection. The core and cladding are shown in figure.

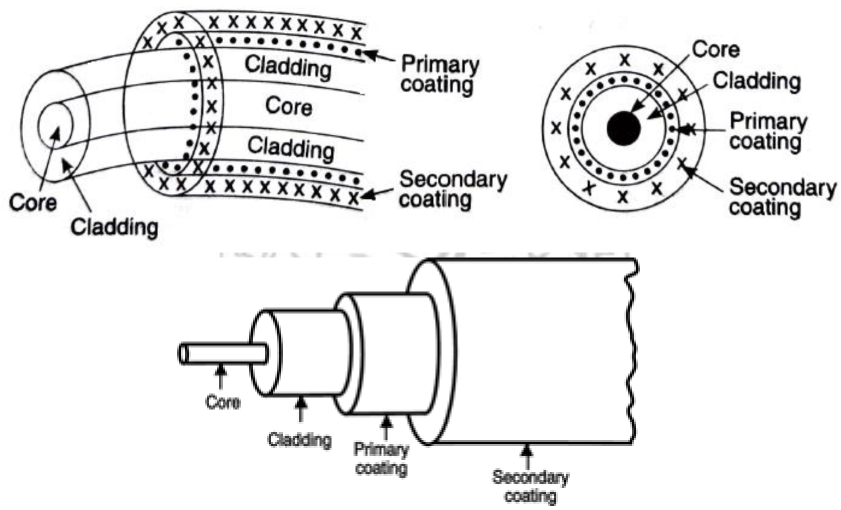


Fig. 4.19

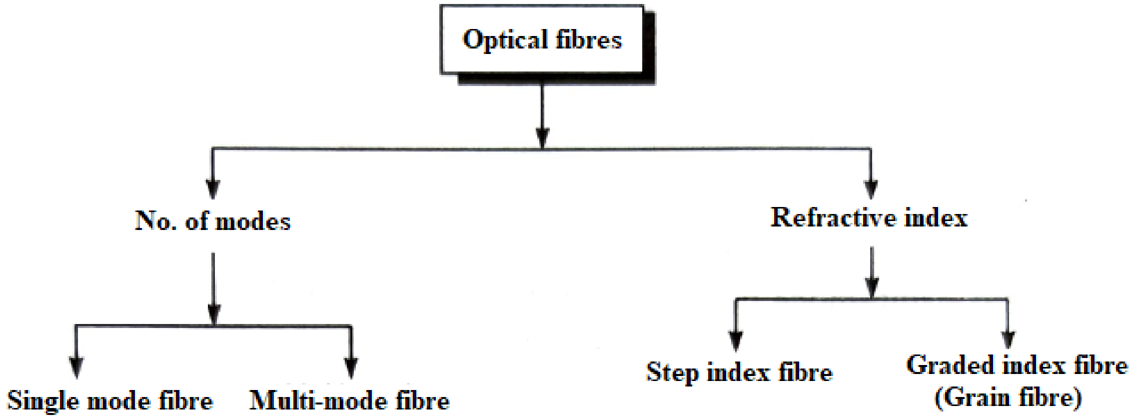
The core diameter can vary from about 5 μm to about 100 μm. The cladding diameter is usually 125 μm. For greater strength and protection of the fibre, a soft plastic coating (primary coating) is done whose diameter is about 250 μm. This is often followed by another layer of coating known as secondary coating.

4.8 TYPES OF OPTICAL FIBRES

The optical fibres are classified into two categories based on:

1. The number of modes, and

2. The refractive index.



4.8.1 CLASSIFICATION OF FIBRE BASED ON NUMBER OF MODES

On the basis of number of modes of propagation, the optical fibres are classified into two types:

(i) Single mode fibre (SMF), and (ii) Multi-mode fibre (MMF).

(i) Single mode fibre (SMF)

The single mode fibre has smaller core diameter (5 μm) and high cladding diameter (70 μm). The difference between the refractive indices of the core and the cladding is very small. When only a single mode is transmitted through an optical fibre, then it is known as single-mode fibre.

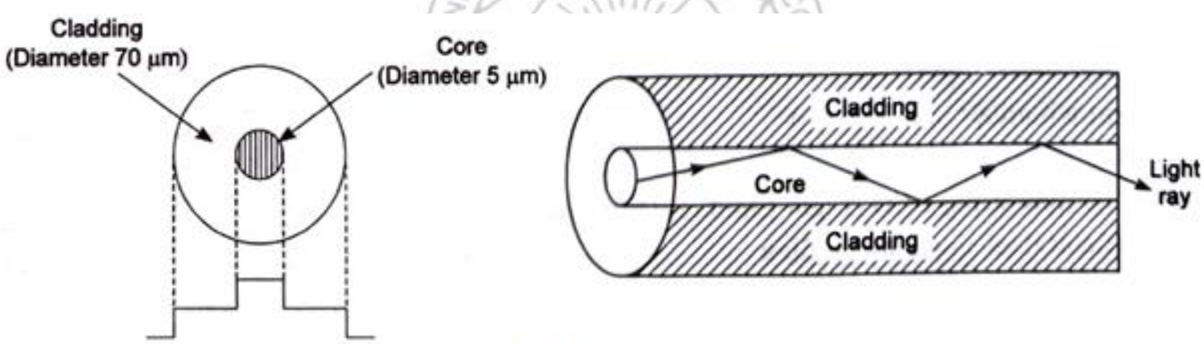


Fig. 4.20

From figure, the light rays can travel only at one discrete path through the core. It can enter and leave the fibre at one angle.

Characteristics

1. The single mode fibre can support only one mode of propagation.
2. Suitable for long distance communication such as telephone lines.
3. The light is passed through laser diodes.
4. Fabrication is very difficult and costly.

(ii) Multi-mode fibre (MMF)

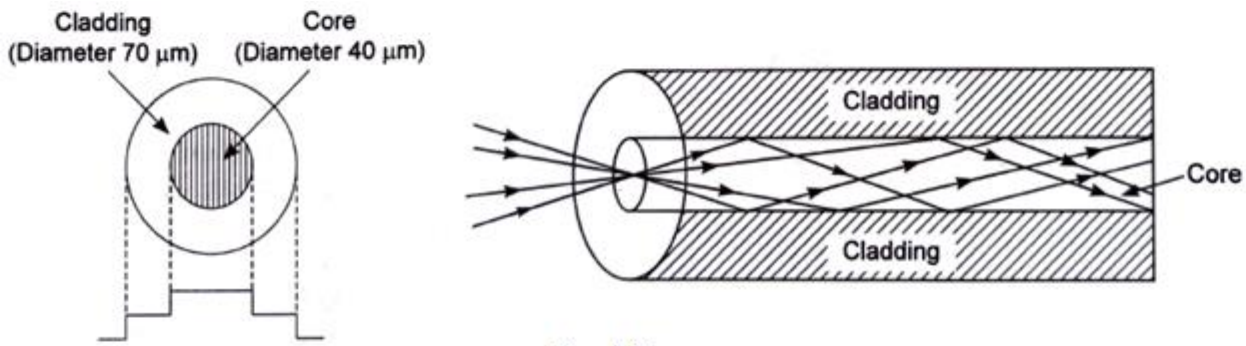


Fig. 4.21

The multi-mode fibre has larger core diameter than single mode fibre. The core diameter is (40 μm) and that of cladding is (70 μm). The relative refractive index difference is also larger than single mode fibre. Multi-mode fibre allows a large number of modes for the light rays through it. When more than one mode is transmitted through an optical fibre, then it is known as multimode fibre. Therefore, in multiomode fibre, the light can travel through the core through many different paths. The light can enter and leave the fibre at various angles.

Characteristics

(i). The multimode can support a number of modes

(ii). Propagation of light is easy

(iii). The light ray enters into the fibre using LED (Light Emitting Diode) source

(iv). The fabrication is less difficult than single mode fibre

(v). The fibre is not costly.

(vi). Not suitable for long distances communication.

4.8.2 CLASSIFICATION OF FIBRE BASED ON REFRACTIVE INDEX

Based on refractive index of the fibre material, there are two types of optical fibres,

(i) Step-index optical fibre, and (ii) Graded-index optical fibre.

(i) Step-index optical fibre

When the refractive indices of core, cladding and air in optical fibre vary step by step, then the fibre is known as step index fibre. It is important to mention here that the core has a uniform refractive index (say n_1), and the cladding has also a uniform refractive index (say n_2), of course $n_1 > n_2$.

Based on refractive index and number of modes, the step index fibres further classified into two types

(a) Step index-single mode fibre and (b) Step index-multimode fibre.

(a) Step index-single mode fibre

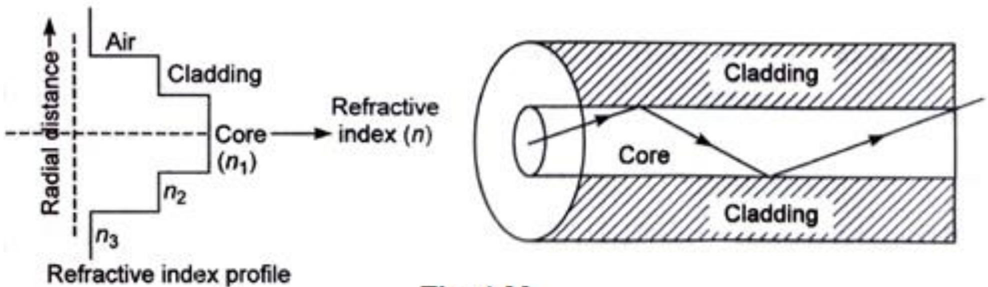


Fig. 4.22

A typical step-index single mode fibre has a core diameter of 5 to 10 μm and an external diameter of cladding of 50 to 125 μm. The core has a uniform refractive index of higher value than the uniform refractive index of cladding. The refractive index profile and light ray propagation in step index-single mode fibre are shown in figure.

(b) Step index-multi mode fibre

In step index-multimode fibre, the core has a much larger diameter, therefore more number of modes of propagation of light can be possible. A typical step-index multimode fibre has a core diameter of 50 to 200 μm and an external diameter of cladding 125 to 300 μm. It has a core material with uniform refractive index and a cladding material of lesser refractive index than that of core.

The refractive index profile and light ray propagation in step-index multimode fibre are shown in figure.

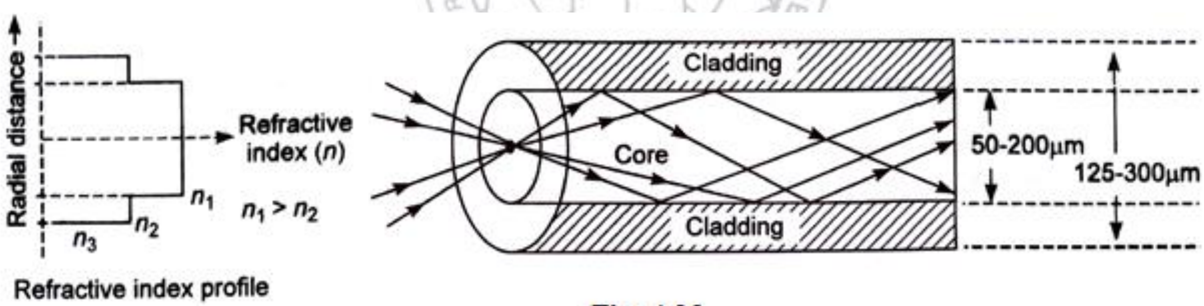


Fig. 4.23

Advantages

Following are the advantages of step-index multimode fibre:

(i) relatively easy to manufacture,

(ii) cheaper than other types,

(ii) larger layer NA,

(iv) they have longer life times than laser diodes.

Disadvantages

They have the following disadvantages:

(i) lower bandwidth,

(ii) high dispersion, and

(iii) smearing of signal pulse.

(ii) Graded-index optical fibre

If the core has a non-uniform refractive index that gradualy decreases from the centre towards the core-cladding interface, the fibre is called a graded-index fibre. The cladding has a uniforn refractive index. The core and cladding diameters are about 50 μm and 70 μm respectively in case of multimode fibre.

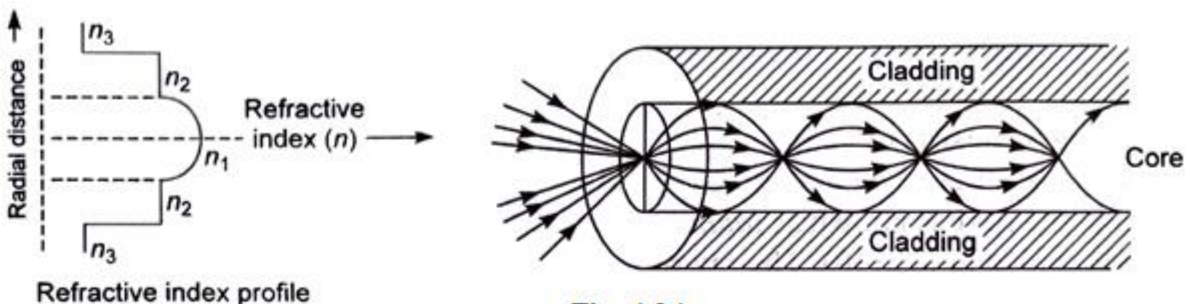


Fig. 4.24

The light rays propagate through it in the form of skew rays or helical rays. They do not cross the fibre axis at any time and are propagating around the fibre axis in helical or spiral manner. The refractive index profile and light ray propagation in graded index fibre (GRIN fibre) are shown in figure.

It is obvious from the figure that a ray is continuously bent and travels a periodic path along the axis. The rays entering at different angles follow different paths with the same period, both in space and time. Thus, there is a periodic self-focusing of the rays.

Advantages

Following are the advantages of graded-index multimode fibre

(i) dispersion is low,

(ii) bandwidth is greater than step-index multimode fibre, and

(iii) easy to couple with optical source.

Disadvantages

Following are the disadvantages of graded-index multimode-fibre

(i) expensive and

(ii) very difficult to manufacture

4.9 FIBRE MATERIALS

The following requirements must be satisfied for selecting a material as optical fibre:

1. It must be possible to make a long, thin and flexible fibres from the material.

2. For the fibre to guide light efficiently, its material must be transparent at a particular optical wavelength.
3. Materials having slightly different refractive indices for the core and cladding must be available.

Glasses and plastics satisfy the above requirements. Therefore, the following combinations of materials are used to construct optical fibres:

(i) Glass core and glass cladding,

(ii) Glass core and plastic cladding,

(iii) Plastic core and plastic cladding.

The majority of fibres are made of glass consisting either of silica or a silicate. To produce two similar materials having slightly different indices of refraction for the core and cladding, either fluorine or various oxides such as B_2O_3, GeO_2 or P_2O_5 are added to silica. It is important to mention here that addition of GeO_2 or P_2O_5 increases the refractive index whereas the doping of silica with fluorine or B_2O_3 decrease the refractive index.

The examples of glass fibres combinations are

	Core	Cladding
1.	$GeO_2 - SiO_2$	SiO_2
2.	$P_2O_5 - SiO_2$	SiO_2
3.	SiO_2	$B_2O_3 - SiO_2$
4.	$GeO_2 - B_2O_3 - SiO_2$	$B_2O_3 - SiO_2$

Here GeO_2-SiO_2 denotes a GeO_2 doped silica glass.

Plastic optical fibre (POF) is a low cost and easy to use fibre for short distance applications like local area network. The fibres have a diameter of around one millimetre while the glass fibres have diameter about (50 μm -62 μm). Thus, the diameter of plastic fibre is far larger than those of glass fibre. Therefore, it is easier to connect critical alignments with plastic fibres. The base material in POF are polymethyl-methacrylate (PMMA), teflon, polysterene, silicon resins and fluoropolymers. These make POF more durable and flexible than glass fibre. The drawback is that they limit its use to about 100 metre. The reason is substantially higher attenuation.

4.10 PROPAGATION MECHANISM AND COMMUNICATION IN OPTICAL FIBRE

The most important application of optical fibre occurs in the field of communication. This is similar to traditional communication system. The purpose is to transfer information from a Source to a distant User.

Principle

The basic principle of optical fibre communication is transmission of information from one place to another place by the propagation of optical signal through optical fibres. It involves deriving an optical signal from electrical signal at transmitting end and converting optical signal back to electrical signal at receiving end.

A basic optical fibre communication system consists of a transmitter which transforms an electrical system (information signal) to be transmitter into an optical signal, a fibre transmission line which conducts the optical signal from transmitted to receiver and a receiver which converts the optical signal back to original signal. These parts are shown in figure.

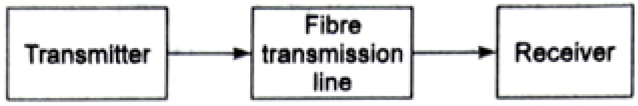


Fig. 4.25

Block diagram of fibre optic communication system

A block diagram of a fibre optic communication system is shown in fig.

The functions of different parts are discussed below:

1. Information system source or message origin

The information signal source may be voice, music, video signals, digital data, etc. which is in analog form to be transmitted. This is converted from analog signal to electrical signal.

2. Transmitter

The transmitter consists of a drive circuit and a light source. The drive circuit transfers the electric input signals into digital pulses. The light source converts the digital pulses into optical pulses. The light source usually used in LED (light emitted diode) or Laser.

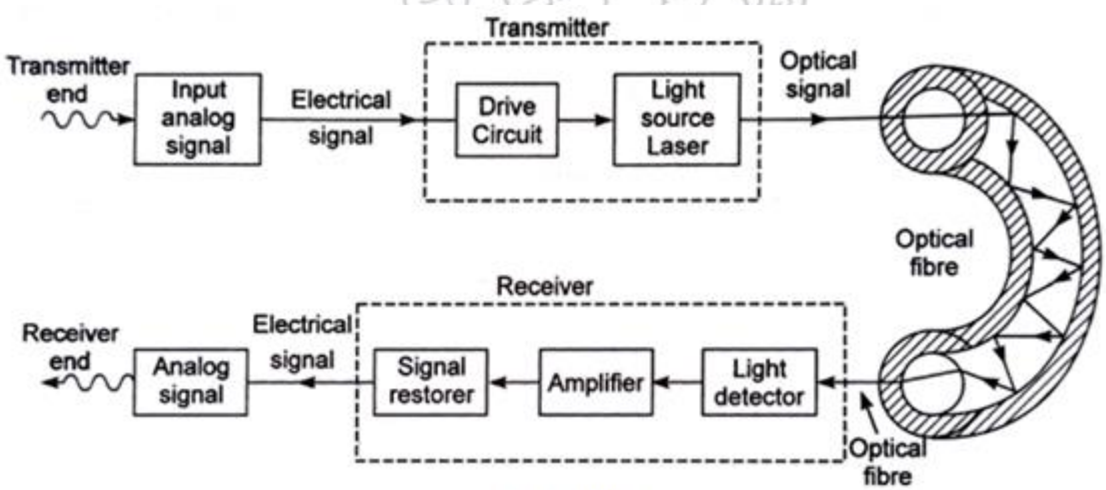


Fig. 4.26

3. Optical fibre

The optical fibre is used as propagation medium. It acts as a wave guide and transmits the optical pulses towards the receiver by the principle of total internal reflection.

4. Receiver

It consists of a photo or light detector, an amplifier and a signal restorer. The photo detector is a receiver which receives the optical pulses and converts into electrical pulses. The amplifier amplifies the signal strength. The amplified electrical signal is converted from digital signal to analog signal with the help of signal restorer.

It this way, the original electrical signal is obtained in analog form with same information.

4.11 ADVANTAGES OF COMMUNICATION WITH OPTICAL FIBRE

- ✓ The optical fibre communication has far more information carrying capacity.
- ✓ Smaller size and weight of the systems make fibre optics more suitable in space and aeronautical applications.
- ✓ Since, fibre is made up of glass hence the raw material of the glass (silica) is available in plenty on the earth.
- ✓ Optical fibres (glass or plastic) are insulators. No electric current flows through them, either due to transmitted signal or due to external radiation striking the fibre. In addition, the optic wave within the fibre is trapped, so none leaks out during transmission to interfere with signals in other fibres.
- ✓ It has lower cost off cables per unit length compared to that of metal counterpart.
- ✓ Light cannot couple into the fibre from outside. Thus, a fibre is well protected from interference and coupling with other communication channels.
- ✓ Fibres will not pick up or propagate electromagnetic pulses caused by nuclear explosions.
- ✓ When high voltage lines are present, a wire communication link could short circuit the lines by falling across them, causing considerable damage. This problem disappears with fibres.
- ✓ As the fibres do not radiate energy within them, hence it is difficult for an intruder to detect the signal being transmitted. So they offer high degree of security and privacy.
- ✓ They can withstand extreme temperature before deteriorating. Temperature approaching 800°C leaves glass fibre unaffected.
- ✓ Corrosion due to water or chemicals is less severe for glass than for copper.
- ✓ There is no need for additional equipment to protect against grounding and voltage problems.

Table: Difference between single mode fibre and multi-mode fibre

S.No	Single mode fibre	Multi-mode fibre
1	Only one mode can propagate through the fibre.	A large number of modes or paths for the light rays may pass through the fibre.
2	The core has smaller diameter and difference in refractive index of core and cladding is very small.	The core diameter is large and the refractive index difference between core and cladding is larger than single mode fibre.
3	There is no dispersion, i.e., degradation of signal during travel in fibre.	There is more dispersion, i.e., degradation of signal due to multimode.
4	Fabrication is difficult and costly.	Fabrication is less difficult and not costly.
5	The fibre can carry information to longer distances.	Information can be carried to shorter distances only.

.**Table: Difference between step index and graded index fibres**

S.No	**Step index fibre**	**Graded index fibre**
1	The refractive index of the core is uniform throughout and undergoes an abrupt step change at cladding boundary.	The refractive index of the core is made to vary gradually such that it is maximum at the center.
2	The path of light propagation is in zig-zag manner.	The path of light propagation is helical, i.e., spiral manner.
3	Distortion is more in multimode step-index fibre.	Distortion is less.
4	Numerical aperature is more for multimode step index fibre	Numerical aperture is less.

UNIT-V

LASERS

5.1 INTRODUCTION

The word LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. Laser is a device that amplifies or increases the intensity of light and produces highly directional light. Laser emits light through a process called stimulated emission of radiation which further amplifies or increases the intensity of light. Some lasers generate visible light but others generate ultraviolet or infrared rays which are invisible.

Einstein gave the theoretical basis for the development of laser in 1917, when he predicted the possibility of stimulated emission. In 1954, C.H. Townes and his co-workers put Einstein's prediction for practical realization. They developed a microwave amplifier based on stimulated emission of radiation. It was called as MASER (Microwave Amplification by Stimulated Emission of Radiation. Maser operates on principles similar to laser but generates microwaves rather than light radiation.

In 1958, C.H. Townes and A. Schawlow extended the principle of masers to light. In 1960, T.H. Maiman built the first laser device.

Laser light is different from the conventional light. Laser light has extra-ordinary properties which are not present in the ordinary light sources like sun and incandescent lamp.

The most important features of lasers are:

(i) high degree of coherence

(ii) high directionality

(iii) extraordinary monochromaticity

(iv) high intensity

In conventional light sources, excited electrons emit light at different times and in different directions so there is no phase relation between the emitted photons. On the other hand, the photons emitted by the electrons of laser are in same phase and move in the same direction. So unlike other sources, lasers produce highly directional, monochromatic, coherent and polarized light beam.

<u>(i) high degree of coherence</u>

We know that visible light is emitted when excited electrons (electrons in higher energy level) jumped into the lower energy level (ground state). The process of electrons moving from higher energy level to lower energy level or lower energy level to higher energy level is called electron transition.

In ordinary light sources (lamp, sodium lamp and torch light), the electron transition occurs naturally. In other words, electron transition in ordinary light sources is random in time. The photons emitted from ordinary light sources have different energies, frequencies, wavelengths, or colors. Hence, the light waves of ordinary light sources have many wavelengths. Therefore, photons emitted by an ordinary light source are out of phase.

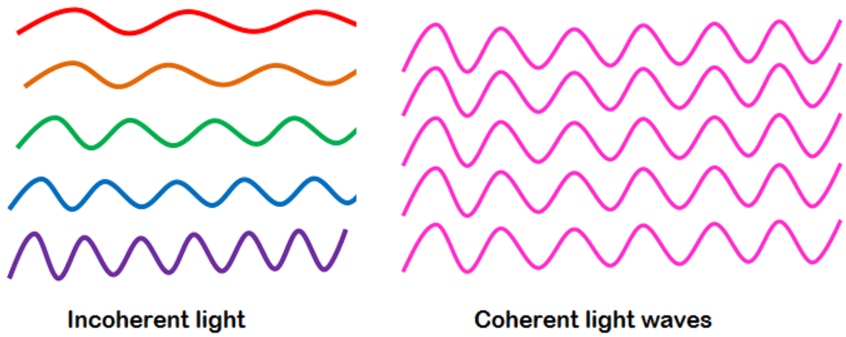


Fig. 5.1

In laser, the electron transition occurs artificially. In other words, in laser, electron transition occurs in specific time. All the photons emitted in laser have the same energy, frequency, or wavelength. Hence, the light waves of laser light have single wavelength or color. Therefore, the wavelengths of the laser light are in phase in space and time. In laser, a technique called stimulated emission is used to produce light.

Thus, light generated by laser is highly coherent. Because of this coherence, a large amount of power can be concentrated in a narrow space.

Directionality

In conventional light sources (lamp, sodium lamp and torchlight), photons will travel in random direction. Therefore, these light sources emit light in all directions.

On the other hand, in laser, all photons will travel in same direction. Therefore, laser emits light only in one direction. This is called directionality of laser light. The width of a laser beam is extremely narrow. Hence, a laser beam can travel to long distances without spreading.
If an ordinary light travels a distance of 2 km, it spreads to about 2 km in diameter. On the other hand, if a laser light travels a distance of 2 km, it spreads to a diameter less than 2 cm

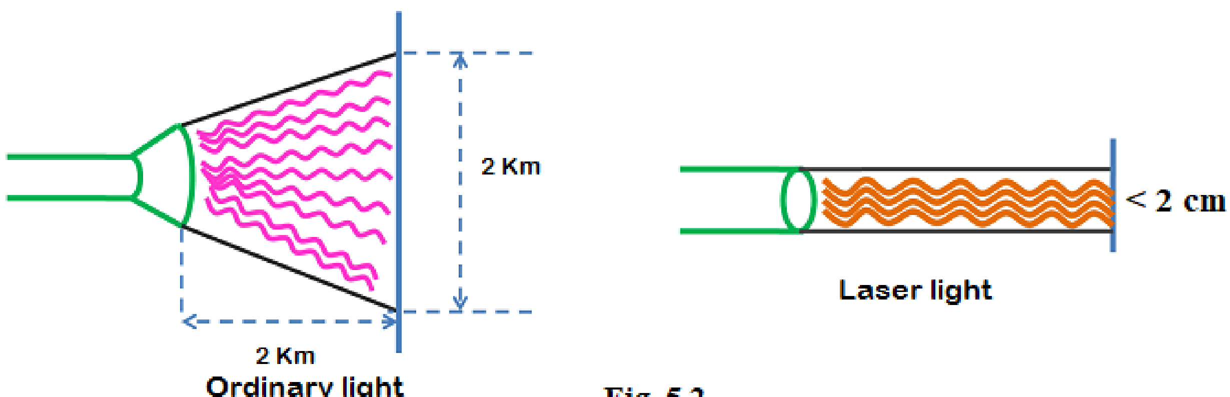


Fig. 5.2

Monochromatic

Monochromatic light means a light containing a single color or wavelength. The photons emitted from ordinary light sources have different energies, frequencies, wavelengths, or colors. Hence, the light waves of ordinary light sources have many wavelengths or colors. Therefore, ordinary light is a mixture of waves having different frequencies or wavelengths.

On the other hand, in laser, all the emitted photons have the same energy, frequency, or wavelength. Hence, the light waves of laser have single wavelength or color. Therefore, laser light covers a very narrow range of frequencies or wavelengths.

High Intensity

You know that the intensity of a wave is the energy per unit time flowing through a unit normal area. In an ordinary light source, the light spreads out uniformly in all directions.
If you look at a 100 Watt lamp filament from a distance of 30 cm, the power entering your eye is less than 1/1000 of a watt.

In laser, the light spreads in small region of space and in a small wavelength range. Hence, laser light has greater intensity when compared to the ordinary light.

If you look directly along the beam from a laser (caution: don't do it), then all the power in the laser would enter your eye. Thus, even a 1 Watt laser would appear many thousand times more intense than 100 Watt ordinary lamp.

5.2 ABSORPTION OF RADIATION

We know that an electron in an atom revolves around the nucleus in discrete orbits. When the atom absorbs sufficient energy by any means in the ground state, the electrons of the atom absorb energy and are excited to higher energy levels. Now the atom is said to be in excited state.

Let us consider two energy levels 1 and 2 of an atom with energies E_1 and E_2 as shown in Fig. 5.3. Let the atom be exposed to light radiation, i.e., a stream of photons with energy **h*ν***. Suppose the atom is initially in lower state 1. The process of atom transfer from normal state (1) corresponding to minimum energy of the system to a higher energy state is termed as excitation. Now, the atom is said to be in excited state. In this process, the adsorption of energy from external field takes place.

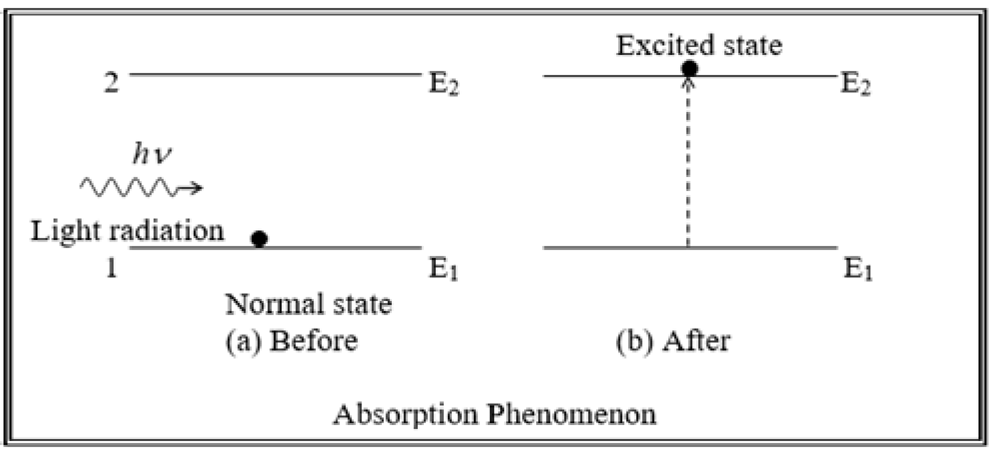


Fig. 5.3

The provided the photon energy h*ν* equals the energy difference $(E_2 - E_1)$. Therefore,

$$h\nu = E_2 - E_1$$

$$\text{or} \qquad \nu = (E_2 - E_1)/h$$

The process is called **stimulated absorption** or simply **absorption.**
Therefore, an atom in ground state with energy E absorbs an incident photon of energy h*ν* and goes to excited state with energy E_2. This process is known as simulated absorption.

Usually the number of excited particles (atoms) in the system in smaller than the non-excited particles (in ground state). Because the time duration which a particle can exist in ground state (normal state) is unlimited while for an excited state it is of the order of 10^{-8} sec. This limited time known as **life time.**

However, there exist some excited states in which the life time greater than 10^{-8} sec. These states are called as meta-states.

5.3 SPONTANEOUS EMMISSION

We know that absorption of a photon of frequency $[\nu = (E_2 - E_1)/h]$, excites the atom from normal state (ground state) E_1 to excite state E_2 which is not a stable state. After a short interval of time (10^{-8} sec), the atom jumps back to ground state by emitting a photon of frequency ν as shown in fig. 5.4. This type of emission is called as spontaneous **emission**.

The emission of radiation from higher energy state **to lower energy state without any external influence is called a spontaneous emission.**

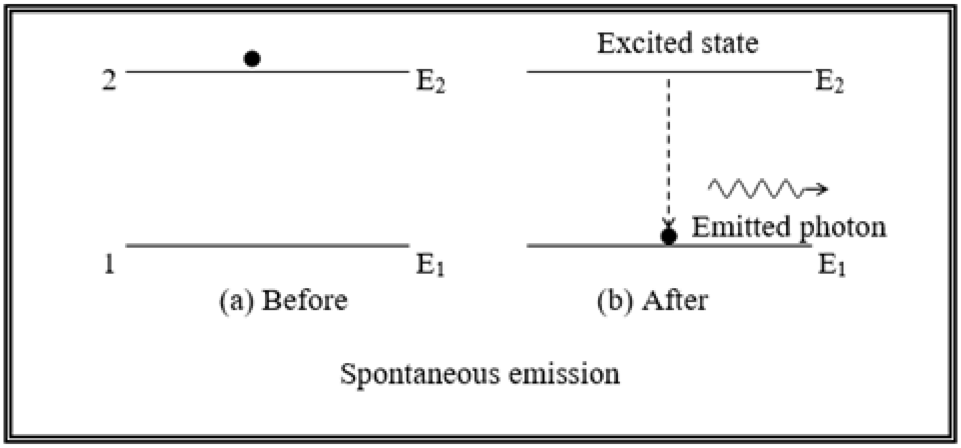


Fig. 5.4

The spontaneous emission is random in character. If there is an assembly of atoms, the radiation emitted spontaneously by each atom has a random direction and a random phase. Thus, radiation in this case is a random mixture of quanta having various wavelengths. The waves neither coincide in wavelength nor in phase. Therefore, the spontaneous emission is *incoherent and has broad spectrum.* So the emission is not much intense.

5.4 STIMULATED (INDUCED) EMISSION

We know that average life time of an atom in the excited state is 10^{-8} sec. During this short interval, let a photon of energy hν is incident on the atom (i.e., when it is still in the excited state) as shown in fig.(5.5). Now the atom jumps to lower energy state, emitting an additional photon of same frequency ν. Hence, two photons move together. This process is called **stimulated emission.**

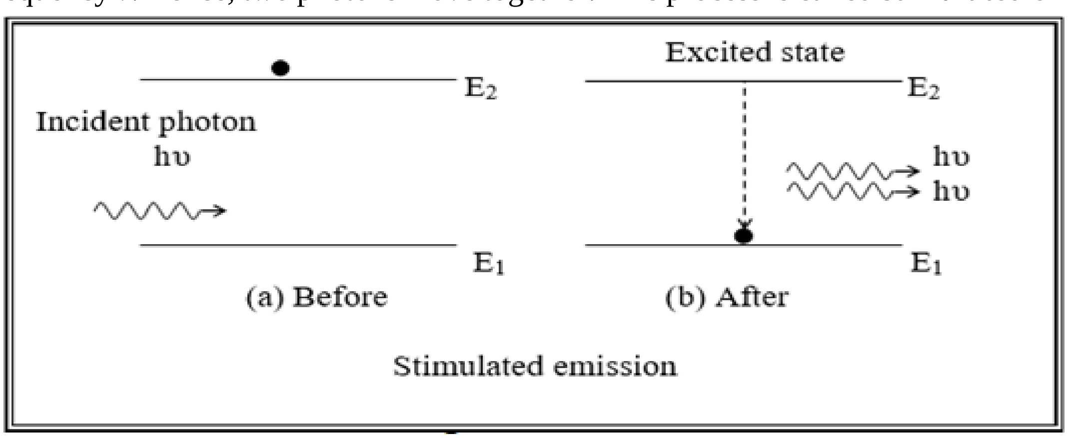


Fig. 5.5

According to Einstein, an interaction between the excited atom and incident photon and trigger the excited atom to make a transition to ground state. The transition generates a second photon which would be identical to triggering photon in respect of frequency, phase and propagation direction .So, in stimulated emission, the emitted wave is of the same frequency and phase as that stimulating incident wave .Their superposition increases the amplitude of the stimulating wave, i.e., there is an amplification. Therefore, the result is an enhanced beam of coherent light.

So, the process of forced emission of photons caused by the incident photon is called stimulated emission.

Difference between spontaneous and stimulated emission:-

Spontaneous emission.	**Stimulated emission.**
1) Emission of light photon takes place immediately during the transition of atom from higher energy level to lower energy level.	1) Emission of light photon takes place by inducement of a photon having energy equal to emitted photon's energy.
2) The emission has a broad spectrum i.e., many wavelengths.	2) The emission has a monochromatic radiation i.e., single wavelengths.
3) Incoherent radiation	3) Coherent radiation
4) Less intensity	4) High intensity
5) Less directionality and more angular spread 6) Ex: Light from sodium (or) mercury vapour Lamp.	5) High directionality and less angular spread during propagation. 6) Ex: Light from laser source.

5.5 EINSTEIN COFFICIENTS

Einstein was the first calculate the probability of transitions assuming the atomic system to be in equilibrium with the incident electromagnetic radiation.

We know that an atom in lower energy state 1 can absorb radiation from incident photon and get excited to higher energy level 2.Thisis known as absorption. Let $\rho(\nu)$ be the energy density of incident radiation, N_1 and N_2 are the number of atoms in the ground and excited states respectively. Now,

in case of stimulated absorption, the probability R_{12} is proportional to

$$R_{12} \propto \rho(\nu)$$

$$R_{12} \propto N_1$$

$$R_{12} = B_{12}\ \rho(\upsilon)\ N_1 \text{ -------- (1)}$$

where B_{12} is Einstein coefficient of stimulated absorption.

Let us consider the transition from state 2 to state 1. In case of spontaneous emission, the probability $(R_{21})_{sp}$ is proportional to

$$(R_{21})_{sp} \propto N_2$$

$$(R_{21})_{sp} = A_{21}\ N_2 \text{ -------- (2)}$$

Where A_{21} is Einstein coefficient of spontaneous emission

N_2 is number of atoms in excited state.

In case of stimulated emission, the rate of stimulated emission $(R_{21})_{st}$

$$(R_{21})_{st} \propto \rho(\nu)$$

$$(R_{21})_{st} \propto N_2$$

$$(R_{21})_{st} = B_{21}\ \rho(\upsilon)\ N_2 \text{ ------- (3)}$$

where B_{21} is Einstein coefficient of stimulated emission.

<u>Relation between different Einstein coefficients:</u>

Under thermal equilibrium, the number of transitions from E_2 to E_1 must be equal to the number of transitions from E_1 to E_2.

$\therefore$ The rate of absorption = The rate of emission

$$R_{21} = (R_{21})_{sp} + (R_{21})_{st}$$

So, in equilibrium condition

$$B_{12}\ \rho(\upsilon)\ N_1 = A_{21}\ N_2 + B_{21}\ \rho(\upsilon)\ N_2$$

$$B_{12}\ \rho(\upsilon)\ N_1 - B_{21}\ \rho(\upsilon)\ N_2 = A_{21}\ N_2$$

$$\rho(\upsilon)\ \left[B_{12}\ N_1 - B_{21}\ N_2\right] = A_{21}\ N_2$$

$$\rho(\upsilon) = \frac{A_{21}\ N_2}{\left[B_{12}\ N_1 - B_{21}\ N_2\right]}$$

$$= \frac{A_{21}}{B_{12}\left[\dfrac{N_1}{N_2} - \dfrac{B_{21}}{B_{12}}\right]}$$

$$\rho(\upsilon) = \frac{A_{21}}{B_{12}\left[\dfrac{N_1}{N_2} - \dfrac{B_{21}}{B_{12}}\right]} \text{ -------- (4)}$$

According to Boltzmann distribution law, the ratio of N_1 and N_2 is given by

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/kT} \qquad \left[\because N = N_0\ e^{-E/KT}\right.$$

$$\left.\Delta E = E_2 - E_1 = h\upsilon\right]$$

$$\frac{N_1}{N_2} = e^{h\upsilon/kT} \text{ --------- (5)}$$

Where k is Boltzmann constant

$$(4) \Rightarrow \because \rho(\upsilon) = \frac{A_{21}}{B_{12}\left[e^{h\upsilon/kT} - \dfrac{B_{21}}{B_{12}}\right]} \text{ --------- (6)}$$

According to plank's radiation formula

$$\rho(\upsilon) = \frac{8\pi h \upsilon^3}{c^3}\left[\frac{1}{\left(e^{h\upsilon/kT} - 1\right)}\right] \text{---------} (7)$$

Compare (6) & (7)

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h \upsilon^3}{c^3} \quad \text{and} \quad \frac{B_{21}}{B_{12}} = 1 \text{----------} (8)$$

$$\because B_{21} = B_{12} = \frac{c^3}{8\pi h \upsilon^3} A_{21} \text{---------} (9)$$

Equation (8) are known as Einstein Relations and equation (9) gives the relationship between A and B coefficients.

Einstein's coefficient of stimulated emission is directly proportional to the cube of frequency of radiation i.e., υ^3 .

5.6 POPULATION IVERSION

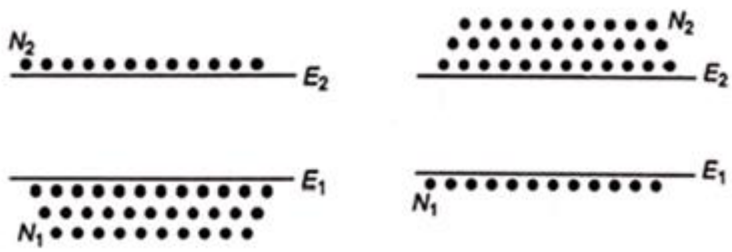
Usually the number of particles N_2, i.e., population of high energy level 2 is less than the population N_1 of low energy level 1. If E_1 and E_2 (E_2>E_1) are two energy states with population N_1 and N_2 then

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/kT}$$

Since $E_2 > E_1$, hence $N_1 > N_2$. In the situation, the system absorbs appropriate electromagnetic radiation incident on it. For laser action to take place, the higher energy levels should be more populated than the lower energy levels, i.e.,$N_2 > N_1$.

The process by which the population of a particular higher energy state is made more than that of a specified lower energy state is called as population inversion.

A system in which population inversion is achieved is called as active system.



(a) normal condition $N_1 > N_2$ (b) Polpulation inversion $N_2 > N_1$

Fig. 5.6

The process of achieving population inversion is known as **pumping** of atoms. The following methods are commonly used for pumping:

1. Optical pumping (used in Ruby laser)
2. Electric discharge (used in Helium-Neon laser)
3. Direct conversion (used in semiconductor laser)
4. Chemical reaction (used in CO_2 laser)

5.7 METASTABLE STATE

We know that normally an atom in the excited state has very short life time which is of the order of 10^{-8} second. Therefore, even if we supply energy continuously to atoms, to transfer them from ground state E_1 to excited state E_2, they immediately come down to ground state. So, in this way, population inversion cannot be achieved. In order to achieve population inversion, we must have energy state which has long lifetime. Such energy state is called as **metastable state.**

The metastable state allows accumulation of large number of excited atoms at this level. Hence, the population inversion can be achieved. Metastable states can be readily obtained in the crystal containing impurity atoms. These levels lie in the forbidden band gap of the host crystal. It is important to mention that the energy levels of the crystal after adding impurity and entirely different from the energy levels of pure crystal.

5.8 PROCESS OF POPULATION INVERSION

Fig. 5.7 shows the process of population inversion in which $N_2 > N_1$.Here, we consider a three level quantum system which has energy levels E_1, E_2 and E_3. Here, $E_3>E_2>E_1$ and E_2 is a metastable state (a state in which the atom stays for unusually long time).

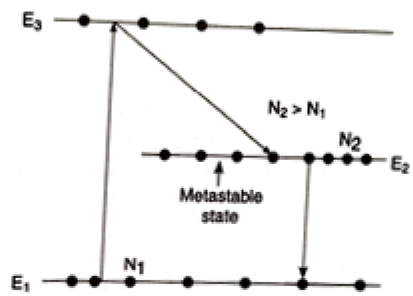


Fig. 5.7

Suppose an appropriate energy from an external source is applied to the system. As a result, some atoms from lower energy state E_1 are excited to higher energy state E_3. Most of excited atoms undergo spontaneous downward transitions to state E_1 while some have transitions to state E_2. We know that the probability of the transition from state E_2 to state E_1 is very low. Therefore, the atoms which go to state E_2 stay there for a long duration. In due course of time, the population of time, the population of E_2 state increase more than the population of E_1 state. Thus, a state is reached when $N_2>N_1$, i.e., population inversion is achieved.

5.9 LASER PRINCIPLE

Let us consider an assembly of atoms of some kind that have metastable states. The basic requirement to the laser action is that there must be more atoms in metastable state than the ground state, i.e., **population inversion of metastable state should be greater than population of ground state.** When this is achieved there will be more stimulated emissions from atoms in metastable state than induced absorption by atoms in ground state. The following steps take place in laser action:

Step 1: Pumping

Fig. (7) shows the energy levels E_0, E_1 and E_2 of an atom of laser medium. Here, E_0 is ground level and $E_2 > E_1$, where E_1 is the metastable energy level.

By supplying energy from the external source, the atom in ground state (E_0) are pumped to excited state E_2.In the optical pumping the laser medium irradiated by radiation of frequency v_0 such that $hv_0 = (E_2-E_0)$.The atoms are excited by stimulated absorption.

Step 2: Population inversion

The atoms from energy level E_2 may drop to metastable level E_1 by spontaneous emission. This occurs almost instantaneously. As E_1 is metastable state, the excited atoms stay comparatively for a longer time. As a result, soon the number of atoms in energy level E_1 becomes much larger than energy level E_0. In this way **population inversion** occurs between energy levels E_1 and E_0.

Step 3: Stimulated emission

It is important to mention here that a photon of energy $h\nu = (E_1-E_0)$ may be emitted due to spontaneous emission from energy level E_1 to E_0.

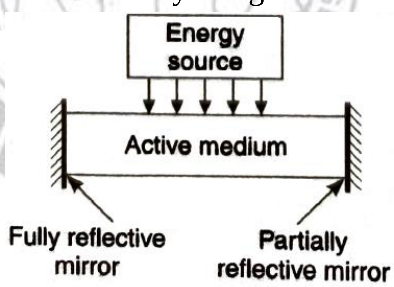
5.10 MAIN COMPONENTS OF A LASER

There are three main components of a laser. There are:

1. **Active medium:** When the active medium is excited, it achieves population inversion. The active medium may be a solid, liquid or gas. Depending on the active medium, we have different types of lasers (ruby), liquid laser and gas laser (He-Ne, CO_2 lasers).
2. **Energy source:** The energy source raises the system to an excited state.
3. **Optical resonator:** The optical resonator consists of two mirror facing each other. The active medium is enclosed by this cavity. Out of two mirrors, one is fully reflective while other is partially transparent.

   The function of the optical resonator is to increase the intensity of laser beam.

   These components are shown schematically in fig.



Action of optical resonator

The action of optical resonator is as follows:

1. Initially, the active centres in non-excited state.

2. Using suitable pumping process, the material is taken into population inversion state. For this purpose, energy source is used.

3. At the initial stage, spontaneous photons are emitted in all directions. The photons that travel in specific direction are selected while others are rejected.

4. The stimulated photons are to be made to pass through the medium a number of times. The mirrors constituting the resonator cause the directional selectivity. The photons travelling in random directions are lost. On reaching the partially reflective mirror, some photons are transmitted out while the remaining are reflected back.

5. The reflected photons de–excite more and more atoms. At fully reflecting mirror, some photons are absorbed while a major number of photons are reflected. The beam is now amplified.

6. The amplified beam undergoes multiple reflections at the mirrors and gains in strength.

7. When the amount of amplified light becomes equal to the total amount of light lost (through the sides of the resonator, through the mirrors and through absorption of the medium), the laser beam oscillation begins. When the oscillation build up to enough intensity, they emerge through front mirror as a highly collimated intense beam, i.e. ., laser light.

5.11 RUBY LASER

<u>Introduction</u>:

In a solid state laser the active medium is crystal in substance. It is the first successful laser achieved by **T. Maiman** in the year 1960. The experimental arrangement are shown in fig. 5.8. It is mainly consists of 3 parts.

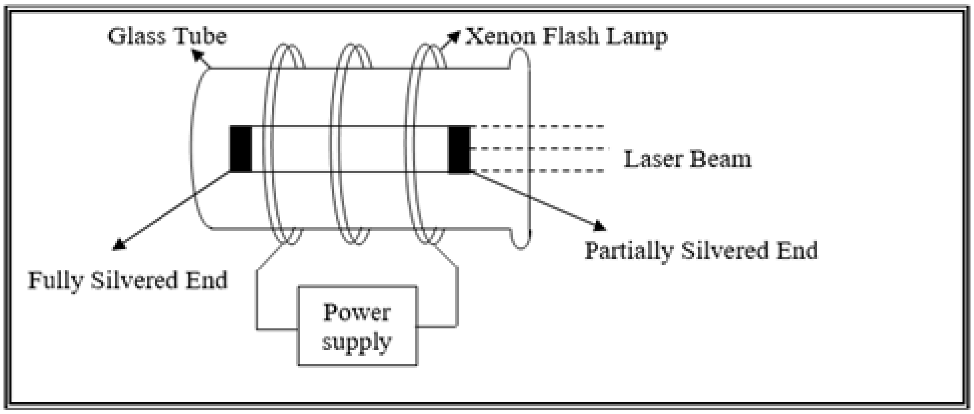


Fig. 5.8

<u>Construction</u>

1) Active working material (Ruby Rod):

Ruby is a crystalline substance of aluminium oxide (Al_2O_3) doped with approximately 0.05% by weight of chromium oxide (Cr_2O_3) the resultant pink colour is due to the presence of Cr^{3+} ions in the appropriate concentration which replace aluminium atoms in the crystal lattice. The length of the ruby rod is 4cm and diameter is 0.5cm.

2) Energy source (xenon tube):

In this laser using optical pumping by helical xenon flash tube that surrounds the tube of ruby rod provides pumping light is rise the chromium ions to the upper energy level. The flash of xenon tube lasts for several thousand milliseconds and the tube consumes several thousand joules of energy. Only a part of energy is used to exit the Cr^{3+} ions and the rest heat up the apparatus. For the purpose a cooling arrangement is used.

3) Resonant cavity:

The end faces of the rod are made strictly parallel grounded and polished to high degree. The end faces and then silvered in such a way that one end face become fully reflected while the other end partially reflecting. Sometimes separate pieces are attached at the end faces.

<u>Working:</u>

The energy level diagram of ruby laser is shown in fig. It represents a 3 level energy of chromium ion. The Cr ion in lower state E_1 are jumped to the excited state E_2 by optical pumping (xenon tube). Where it absorbs a light 5500 A^0.

The excited ions give up by collisions a part of the energy to crystal lattice and decay to the metastable state E_3. This transition is **radiation less transition**.

Now the metastable state has relatively longer lifetime (10^{-3}sec), than that of usual lifetime. So the no of ions in E_2 state goes on increasing. While no of ions is decreasing due to pumping. In this way population inversion is achieved E_2 and E_1.

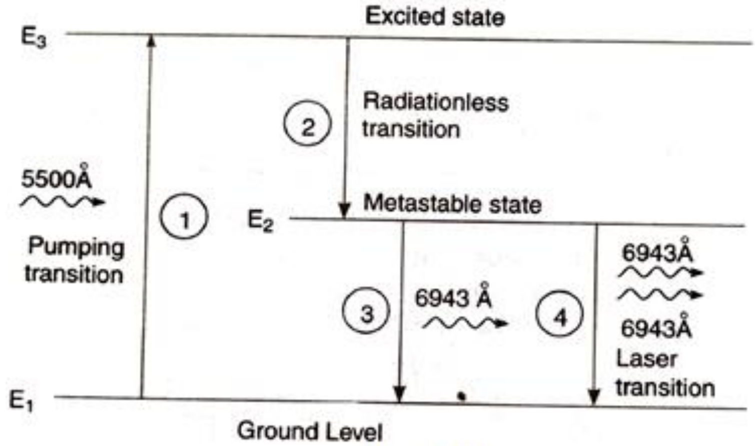


Fig. 5.9

The ion passes spontaneously from metastable state to ground state. It emits radiation of 6943A^0. This photon travels from through ruby rod, and stimulated an excited atom. The excited atom emitting photon and return to ground level. This state transition is laser transmission.

The process is repeated again and again this results amplified strong laser beam is emitted.

Characteristics of Ruby laser:

1. Active medium –Ruby.

2. Pumping-xenon tube.

3. Optical resonator-Highly polishing and silver coating.

4. Wave length of output- 6943A^0.

5. Nature of output-pulsed laser.

Drawbacks:

1. Laser require high power pumping.

2. It is pulsed laser

3. Imperfection of crystal

Applications:

1. It is used in laboratory experiments.

2. It is used in soldering and welding.

3. It is used for drilling of brittle material on a very small area.

4. It is used to test the quality of the material.

5. It is used in the treatment of detached retina.

6. It is used in light detection and ranging (LIDAR).

5.12 HELIUM-NEON LASER

The main drawback of ruby laser is that the output beam is not continuous though very intense. For the continuous laser beam, gas lasers are used. In gas lasers the vapours of metals are employed as active media. The main advantages of gas lasers are exceptionally high monochromaticity, most pure spectrum and high stability of frequency. Hence, thev have wide applications in various branches of science and engineering particularly in communications. The output power of gas lasers is moderate but inferior to that of crystal lasers.

In 1961, A. Javan, W. Bennett and D. Hermiot reported a continuous He-Ne gas laser.

Construction

The experimental arrangement of He-Ne laser is shown in fig. 5.10.

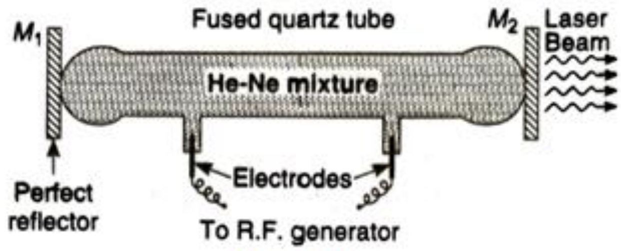


Fig. 5.10

The gas laser consists of a fused quartz tube with diameter of about 1.5 cm and 80 cm long. This tube is filled with a mixture of neon (Ne) under a pressure of 0.1 mm of mercury and helium (He) under a pressure of l mm of mercury. There is a majority of helium atoms and minority of neon atoms. At one end of the tube, there is a perfect reflector while on the other end is a partial reflector. The active material is excited by means of a high frequency generator with a frequency of several tens of MHz and an input of about 50 watt.

Working

The energy level diagram of He-Ne laser is shown in fig. (5.11).

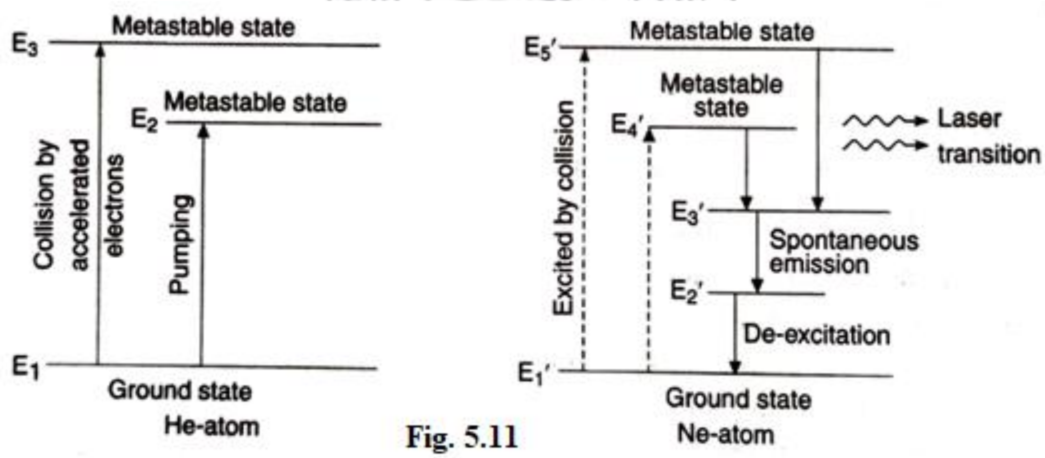


Fig. 5.11

When a discharge passes through the gas mixture, helium atoms are excited to higher energy levels E_2 and E_3 through collisions with accelerated particles. This is termed as pumping. The states E_2 and E_3 are metastable states from which there are no allowed transitions.

The excited helium atoms then collide inelastically with neon atoms still in ground state and transfer energy to them. The advantage of this collision process is that fairly light neon atoms can easily jump to energy states E_3', E_4' and E_5'. It is important to mention here that after collision, the helium atoms are returned to ground state. The higher Ne states $E_4'$and $E_5'$are metastable states and have longer life times than E_3'. Therefore, a population inversion takes place between states E_5', E_4' and E_3'. When an excited Ne atom passes from metastable states E_5' and E_4' to state E_3', it emits a photon. This photon travels through the gas mixture. If the photon is moving parallel to the axis of the tube, it is reflected back and forth by the mirror-ends until it stimulates an excited Ne-atom. Thus, it causes a fresh photon in phase with stimulating photon. The stimulated transition is a laser transition. This process continues till a beam of coherent radiation builds up in the tube. When the beam becomes sufficiently intense, a portion of it escapes through the partially silvered end.

Characteristics of He-Ne laser

1. Type: It is a four energy levels (3 in Ne and 1 in He) laser.

2. Active medium: It uses a mixture of helium and neon gases as the active medium.

3. Pumping method: Electric discharge method is used for pumping action, i.e., for achieving population inversion.

4. Optical resonator: A pair of plane mirrors facing each other is used as optical resonator.

5. Frequency of output: The frequency of output beam is about 4.7 x 10 Hz.

6. Wavelength of output: The wavelength of laser output is 6328 A.

7. Nature of output. The nature of output is continuous waves.

8. Power output: The power output of laser beam is 0.5-50 milliwatt.

Advantages or merits of He-Ne laser

1. This operates in a continuous wave mode.
2. It is more monochromatic and more directional than solid state lasers.
3. It has high stability of frequency.
4. No cooling is required.
5. It is less inexpensive.

Applications and uses of He-Ne laser

1. It is used in laboratory experiments to produce interference and diffraction patterns.
2. It is used in optical communication without fibre for moderate distance.
3. It is used for aligning the ruby laser.
4. It is used in ophthalmology.
5. It can be used to produce holograms, i.e., 3D photographs.

5.13 APPLICATIONS OF LASER

The lasers are put to a number of uses in different branches of science due to their narrow band width and narrow angular spread. A few applications are listed below

(1) Communications

(i) Due to the narrow band width, lasers are used in microwave communication. We know that in microwave communication the signal is mounted on carrier waves by the process of modulation. As the band width of carrier waves is limited, the number of channels of message which can be carried simultaneously is limited. But by the use of lasers, more channels of message can be accommodated because the band width is very small.

(ii) Due to narrow angular spread, the laser beams have become a means of communication between the earth and moon or other satellites. The earth-moon distance has been measured with the use of lasers.

(iii) Laser radiation is not absorbed by water and hence it can be utilized in under-water communication networks.

(iv) Fibre guides. A laser beam in conjunction with optical fibre can be used to transmit audio signals over long distances without attenuation or disturbance.

(2) Computers

By the use of lasers, the storage capacity for information in computers is greatly improved due to narrowness of bandwidth. The IBM Corporation is trying to transmit an entire memory bank from one computer to another by the use of laser beam.

(3) Industry

The lasers have wide industrial applications. Lasers can be focused into a very fine beam, resulting in raising the temperature about 1000 K. So, they can blast holes in diamonds and hard steels.

(4) Medicine

They have wide medical applications. They have been used successfully in the treatment of detached retinas. Preliminary success had also been achieved in treatment of human and animal cancers.

Micro-surgery is also possible because laser beams can be focused on very small areas (due to the narrow angular spread) and hence one harmful component can be destroyed without seriously damaging the neighboring regions.

5) Military Applications

Their study is also oriented for military purposes. Due to high energy density, a laser beam can be used to destroy very big objects like aircrafts, missiles, etc. in a few second by directing the laser beam into the target. As such it is called 'death ray' or 'ray weapon'. Laser beam can be used in laser gun. In a laser gun, highly convergent beam is focused on enemy targets at a short range.

(6) Chemical applications

Lasers have wide chemical applications. They can initiate or fasten certain chemical reactions which could not be possible in the absence of suitable photons. They can be used for investigating the structure of molecules. Raman spectroscopy is one in which lasers have made so much impact that a separate branch named as Laser Raman Spectroscopy has grown rapidly. By the use of lasers, the Raman spectrum can be obtained for much smaller samples and faster too. Not only that but some interactions also arise due to high intensity excitation which provide additional information.

(7) Weather forecasting

Pictures of clouds, wind movements, etc. can be obtained with laser beam. The data so obtained can be used in weather forecasting.

(8) Lasers in photography

Using laser, we can get three dimensional lens less photography. Using interference techniques, we can take hologram which is analogous to negative of the photographic film.

HOLOGRAPHY

5.14.1 Ordinary Photography

The ordinary photograph gives us only a two-dimensional image of the object. In ordinary photography we use lenses to focus the image on a photographic plate. The focusing takes place only in a single plane and all other planes are out of focus. Thus, there is a two-dimensional recording of a three-dimensional object.

The photographic plate records only the intensity variations while the phase distribution at the plane of photographic plate is completely lost. After the development of photographic plate, only two dimensional picture is obtained. When we see the photograph from various directions, we are unable to see what is happening on the other side of the object. Hence, the three dimensional character of the object is lost in recording.

5.14.2 Holography

Holography is a new kind of photography where the image of the object is not recorded but the light waves reflected from the object are recorded. The photographic record is called as hologram. The word "holography" originates from the Greek word "hole" meaning the whole and "graphy" meaning the writing, i.e., holography means "complete recording".

It should be noted that the holograph has no resemblance with the object, of course, it contains all information about the object in kind of optical code. When the holograph is illuminated by coherent source of light, a three dimensional image of the original object is formed. The process of image formation from hologram is known as reconstruction process.

So, holography is a two-step process:

1. Transformation of the object into hologram, i.e., an object illuminated by coherent light is made to produce interference fringes in a photographic emulsion, and
2. Retransformation or reconstruction of hologram into the image of the object, i.e., re-illumination or the developed interference pattern by light of same wavelength to produce a three dimensional image of the original object.

5.14.3 GABOR HOLOGRAM

The principle of holography was first put forward by Dennis Gabor when he began conducting his famous experiment in holography at the Research Laboratory of British Thomson-Houston Company. It was a two-step lens less imaging process. The first step was the photographically recording of an interference pattern. The pattern was generated by the interaction of scattered quasi-monochromatic light from an object and coherent reference wave. He called the pattern as hologram.

An object O is embedded in a parallel beam of coherent light which falls on a photographic plate. The diffracted light from the object O superposes on the coherent incident beam and produces a hologram on the photographic plate. At each point of the photographic plate interference occurs between the undisturbed coherent light and the diffraction field of the object. The second step was the reconstruction of the optical image. This was done through the diffraction of the coherent beam from developed hologram. The hologram is illuminated by laser light. The object now reconstructed. In the reconstruction we have a real image as well a virtual image.

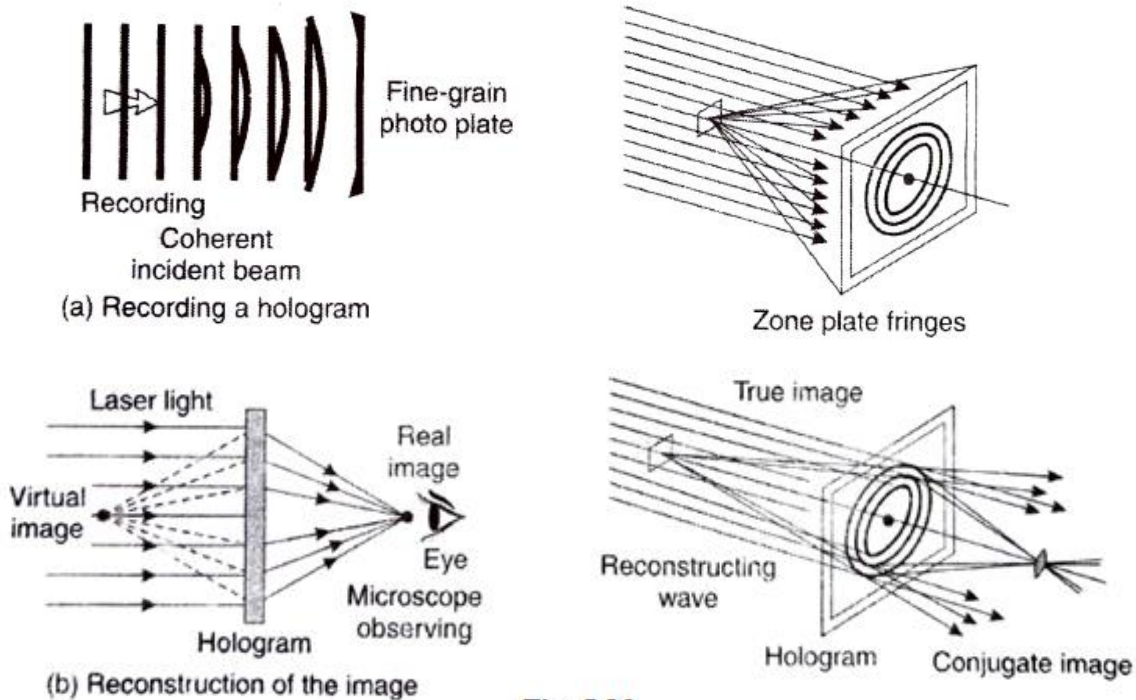


Fig. 5.12

It is important to mention here that the interference pattern (hologram) contains, by the way of fringe configuration, all information corresponding to both the amplitude and phase of the wave scattered by the object. It can be assumed that each point on an extended object generates its own zone plate and ensemble of all such partially overlapping zone plates form the hologram. During the reconstruction step, each constituent zone plate forms both a real and virtual image of single object point. In this way, the hologram forms a complete real and virtual image of the object. The virtual image is sometimes spoken as the true image while the other is the real or conjugate image. Gabor was awarded Nobel Prize in Physics in 1971 for his research in holography.

5.14.4 BASIC PRINCIPLE OF HOLOGRAPHY

When a beam of coherent light is incident on object, it get diffracted and overlapped with the same direct coherent beam. Now interference fringes are produced on a photographic plate placed in front of the object. Consider a point object A is embedded in a monochromatic plane wave incident from left. The scattered wavefronts are spherical with centre at the point object. It should be remembered that the scattered wavefronts are characterized by their amplitudes and phases. The scattered or diffracted light from the object A superposes over the coherent incident light and produces a hologram on the photographic plate P, placed on right side of A. Due to constructive and destructive interference between diffracted light and direct reference beam, the pattern is of bright and dark concentric circles on photographic plate.

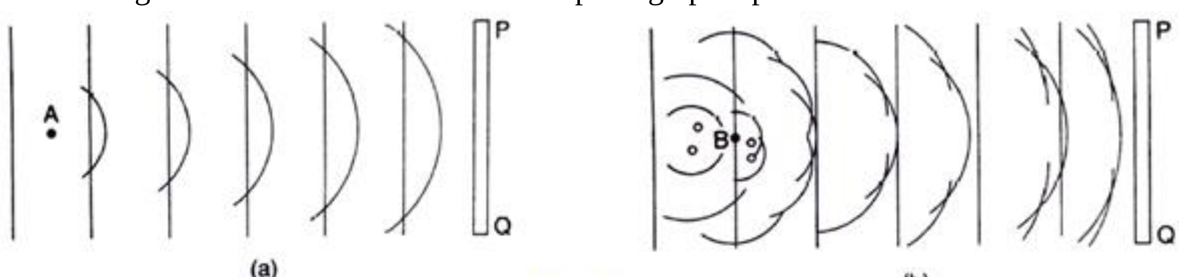


Fig. 5.13

Now consider the case of a three-dimensional solid B. The solid may be regarded as a collection of a number of points. Each point diffracts its own set of spherical waves. The resultant wave pattern diffracted from the entire solid is very complex. Now the interference between diffracted wavefronts and reference beam modifies intensity at all points. The modification in intensity is in accordance with phase relationship between the waves at all points. In this way the phase variation in the wave pattern diffracted from the object are converted into intensity variations. The interference pattern is formed on photographic plate PQ.

When the photo-plate or hologram is illuminated by laser light, the object is reconstructed.

5.14.5 RECORDING OF A HOLOGRAM

Principle involved in holography is based on interference technique and hence light waves of high degree coherence are required for its realization. So a laser beam is used for this purpose. The following figure shows the arrangement for recording a hologram.

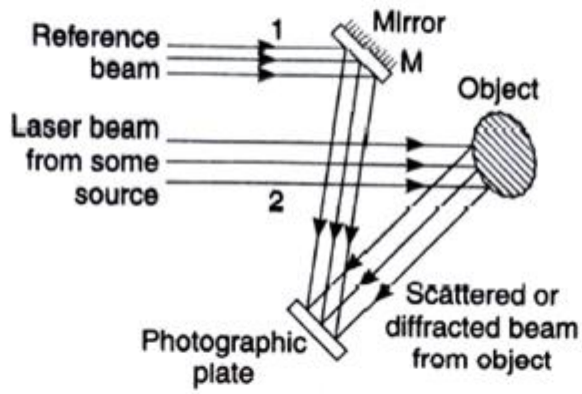


Fig. 5.14

First of all the laser beam is divided into two parts (1 and 2). The second beam illuminates the object and the diffracted or scattered beam falls on the photographic plate P. The first beam (reference beam) is reflected on to the photographic plate by means of a plane mirror M. In this way, the film is exposed to reference beam and diffracted beam simultaneously. Since, both beams belong to same laser wavefront, the beams interfere on the plate. Thus, we obtain a complicated interference pattern on the film. This film is called a hologram. The hologram consists of numerous points making up the image on photographic plate. The hologram doesn't hints the image embedded in it directly but it contains information about the amplitude and also phase of the object wave.

5.14.6 RECONSTRUCTION OF IMAGE FROM HOLOGRAM

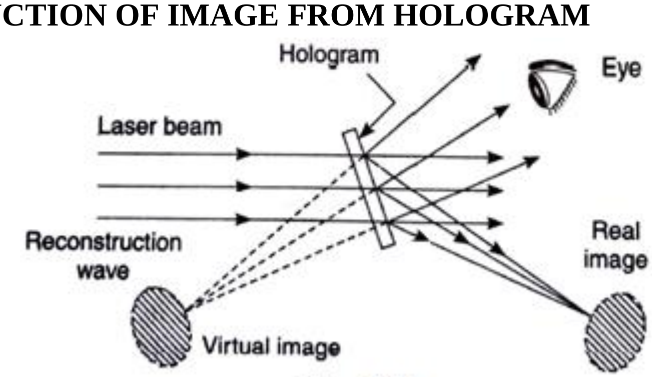


Fig. 5.15

The figure shows the arrangement to reconstruct the image from the hologram. This is a reverse process of making a hologram. The hologram is illuminated by a single beam from laser. The beam is identical to the reference beam used during the formation of hologram. The hologram now acts as a complex grating and diffracts the light. So, in the direct direction of the beam, we get zero order (like diffraction grating) giving no information. The reason is that the laser beam passed through the hologram has only amplitude variation but no information about phase variation. In other directions, the waves diffracted through the hologram carry the phase and amplitude of the waves originally diffracted by the object when the hologram was made. The object wavefronts have thus been reconstructed. Here, it is important to note that one of the diffracted beams forms virtual image as real image, while another forms a conjugated image shown in figure. By moving the position of our eye we can see different perspective of object in the image. The real image can be photographed without the aid of lenses just by placing a light sensitive medium at the position of real image. The virtual image has all the characteristics of the object like parallax, etc.

5.14.7 APPLICATIONS OF HOLOGRAPHY

The principle of holography finds applications in many diverse fields. A few of them are

(1) Holographic Interferometry

One of the most important applications of holography is in the field of interferometry. Using a Special technique known as "double exposure technique", the surfaces that undergo deformations due to stress can be studied.

According to this technique, the photographic plate is first of all partially exposed to the reference beam and object beam as usual. Afterwards, the object is allowed to undergo the deformation by stress and then, the photographic plate is exposed to the new object beam along with the same original reference beam. In this way, the photographic plate is allowed to two different exposures. Now the photographic plate after development forms the hologram. The hologram is once again exposed to reconstruction beam. From the hologram, we get two object waves: one corresponds to the unstressed object and the other to the stressed object. The two waves interfere and produce interference fringes. The quantitative study of these fringe pattern gives the distribution of strain in that object.

(2) Acoustic Holography

Acoustic holography is a technique used to study the image formed by sound waves. This technique is used to view the image of an object in water. For this purpose we use ultrasound waves in place of laser beam to generate reference beam as well as object beam.

An ultrasonic generator is kept inside the water. It generates reference beam that is directed towards underwater object. The ultrasonic waves reflected from the object form the object beam. The reference beam and object beam interfere and produces ripples on the still surface of water. The ripple pattern forms the hologram and can be photographed. Then ripple pattern may be illuminated by laser beam and can be viewed through a telescope.

The acoustic holography can also be used to get a three dimensional image of our internal organs. From the developed hologram, we can reconstruct the three dimensional image from which we can study in depth and the details of the concerned organ.

(3) Holographic Microscopy

Using holography we can also get information about depth. In holographic microscopy, one beam after passing through specimen and microscope, combines with reference beam and produces the hologram. The reconstruction image can be seen in any cross-section desired. Thus, this method provides a high depth of field compared to conventional high power microscope.

The holographic microscopy also finds its applications in the study of time varying phenomena that occur in a certain region. This is not possible with ordinary microscopic techniques. Here, a hologram recorded of the scene as and when it occurs. Thus, the event gets preserved in the hologram. Now, at our convenience, we can focus through the depths of the reconstructed image and study the phenomena in detail. Thus, we can study the transient microscopic events with the help of a hologram.

(4) Other Applications

The other applications include holographic cinema, spatial filtration and character recognition. Long distance holography using microwaves, rainbow holograms, focused image holograms, holographic optical elements, etc.

ALL THE BEST